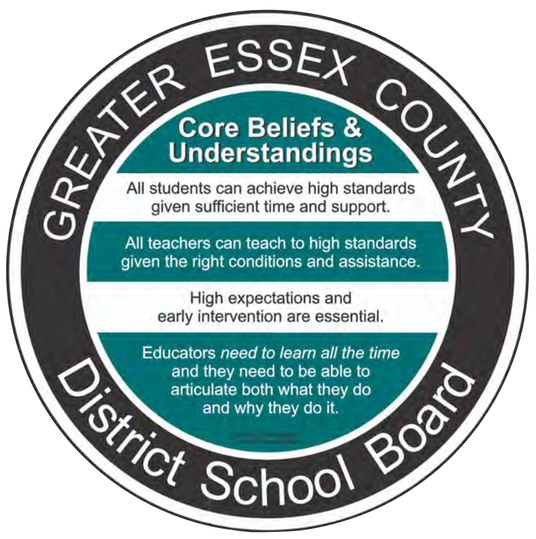
GREATER ESSEX COUNTY DISTRICT SCHOOL BOARD

Experiencing Mathematics From Concrete to Abstract

Our youngest students come to school as curious learners whose life experiences inform their interests. With logic and reasoning, children make sense of the world around them through exploration, inquiry, play and collaboration. Four and five year olds demonstrate an innate ability to make connections and see patterns; in essence, young learners are able to see mathematics in the world around them through a sense of quantity, fairness, the recognition of patterns and rules. Mathematics is a subject where educators allow critical thinking to flourish. Students are encouraged to mathematize their world, and as facilitators of learning, educators help students represent their thinking more abstractly over time using the language of mathematics. Mathematics will foster the types of thinkers who will be vital to our world. What society needs is problem solvers. The GECDSB is committed to creating a learning environment that develops mathematical proficiency for all students, where thinking is encouraged in a mathematics classroom that promotes equity. One way to unlock this potential is making mathematics concrete.



> *"The Greater Essex County District School Board provides mathematics education that engages and empowers students through collaboration, communication, inquiry, critical thinking, and problem–solving, to support each student's learning and nurture a positive attitude towards mathematics."*
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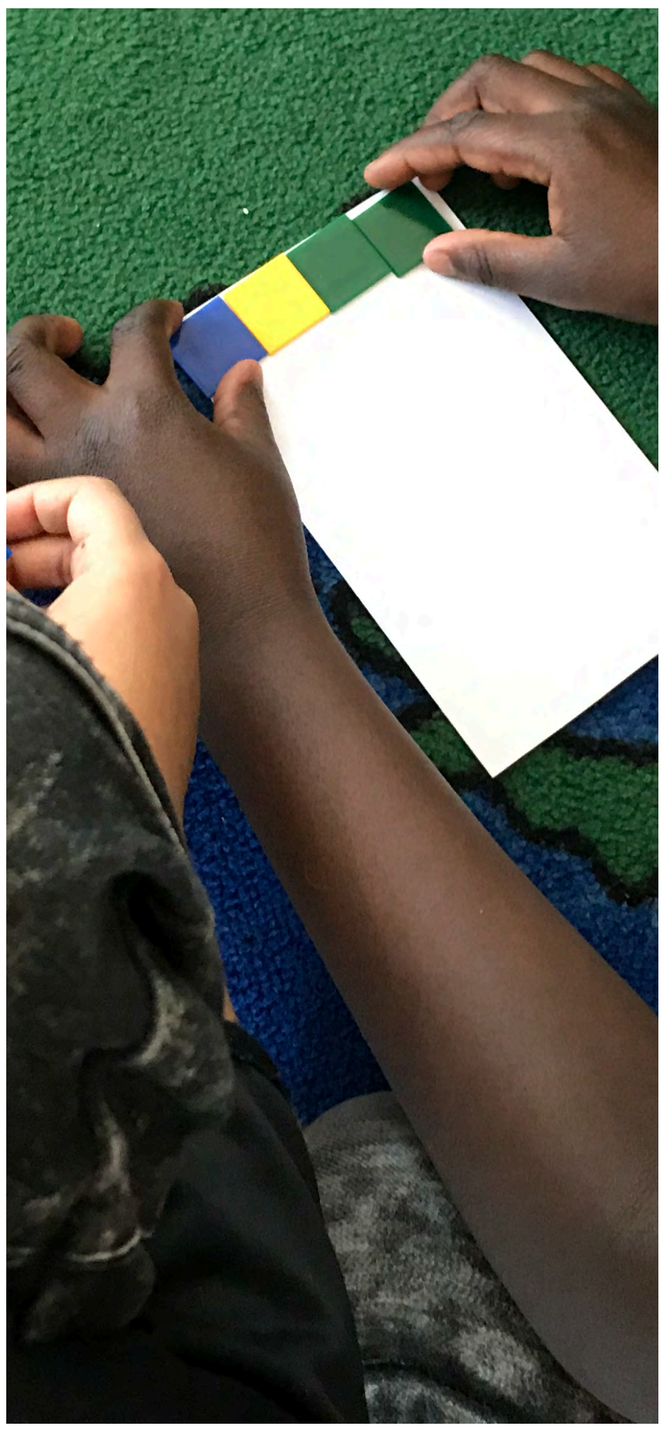
> GECDSB, A Vision for Mathematics, 2016

The purpose of these learning briefs is to share the research, discussion and insights garnered from the intensive work of the Greater Essex County District School Board-Math Task Force. These papers are rooted in the GECDSB core beliefs and the Ontario Mathematics Curriculum, 2005. They are meant to elevate, enrich and extend the discourse of mathematics education with the intention of encouraging a positive and productive disposition toward mathematics.

Each paper provides a list of sources and resources to extend the professional conversation, and enhance the learning. In addition, live links appear at the end of the papers, with connections to various resources.



MATH TASK FORCE 2.0

Making Math Concrete

GECDSB has made a commitment to make math "concrete". Initial professional development was rooted in the research of the CRA (Concrete – Representation – Abstract) model. This model was developed to support students struggling in mathematics, particularly students with a designated learning disability. "CRA is a sequential three level strategy promoting overall conceptual understanding, procedural accuracy and fluency by employing multisensory instructional techniques when introducing the new concepts. Each level builds on the concepts taught previously." (Witzel, Riccomini & Schneider, 2008). CRA was a great starting point for teachers at the GECDSB. It was a way for educators to begin reconsidering how concepts were introduced and how to develop proficiency. Many teachers began asking themselves how to introduce the concept concretely with manipulatives. "A carefully planned mathematics environment enables the use of manipulatives whether commercial products or found objects, sometimes brought in by the students themselves. Ideally, manipulatives serve as learning tools to help students build their understanding and explain their thinking to others. Research has shown, however, that manipulatives themselves do not magically carry mathematical understanding. Rather, they provide concrete ways for students to give meaning to new knowledge." (Ontario Ministry of Education, 2011, p. 2)

Consider for a moment the procedure for calculating mean, a measure of central tendency. Traditionally, this concept was taught as a series of steps. Take all of the numbers in the data set, add them together, and divide by the number of numbers. This procedure seems relatively straightforward. However, in 2017, the Education Quality And Accountablity Office (EQAO) Grade 6 Assessment of Mathematics presented students with the problem shown in figure 1.

Although the procedure for calculating the mean is relatively easy to memorize, students performed poorly on this question. At the GECDSB, 46% of students answered this question correctly. This question is an example of the necessity to introduce a concept concretely in order to develop conceptual understanding. It is difficult for students to memorize and work flexibly within a procedure, if they never really understood it beyond a series of steps that they were asked to follow.

13 This table shows the heights of 6 players on a volleyball team.

Player Height

Player	Height (cm)
Chinua	138
Zaki	132
Baina	116
Bohumir	138
Nura	126
Leslie	130
Peter	?

If the mean height of all 7 players is 128 cm, which of the following is Peter's height?

- ○ 138 cm
- ○ 131 cm
- ○ 130 cm
- ○ 116 cm

Figure 1: Question 13 from the 2017 EQAO Grade 6 Assessment (EQAO Grade 6 Assessment Release Questions, 2017, p. 10)

Our understanding of "concrete" has evolved over the past few years to not only include the use of manipulatives, but to an expanded focus on the use of both concrete and visual representations. "Visual understanding is incredibly powerful for students, adding a whole new level of understanding… This can be provided through diagrams but also through physical objects, such as multilink cubes and algebra tiles" (Boaler, 2016). Leveraging the use of a variety of visual models has helped educators and students bridge their logical reasoning and sense making with representations that are more abstract.

Students in grade 2 determined how many cartons of eggs to purchase in order to have 60 eggs all together. They made sense of this problem using a concrete or visual representation (figures 3 and 4) that displayed one carton of 12 eggs at a time, and they counted on by ones and twos until they had 60 eggs in

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(GECDSB Principal)

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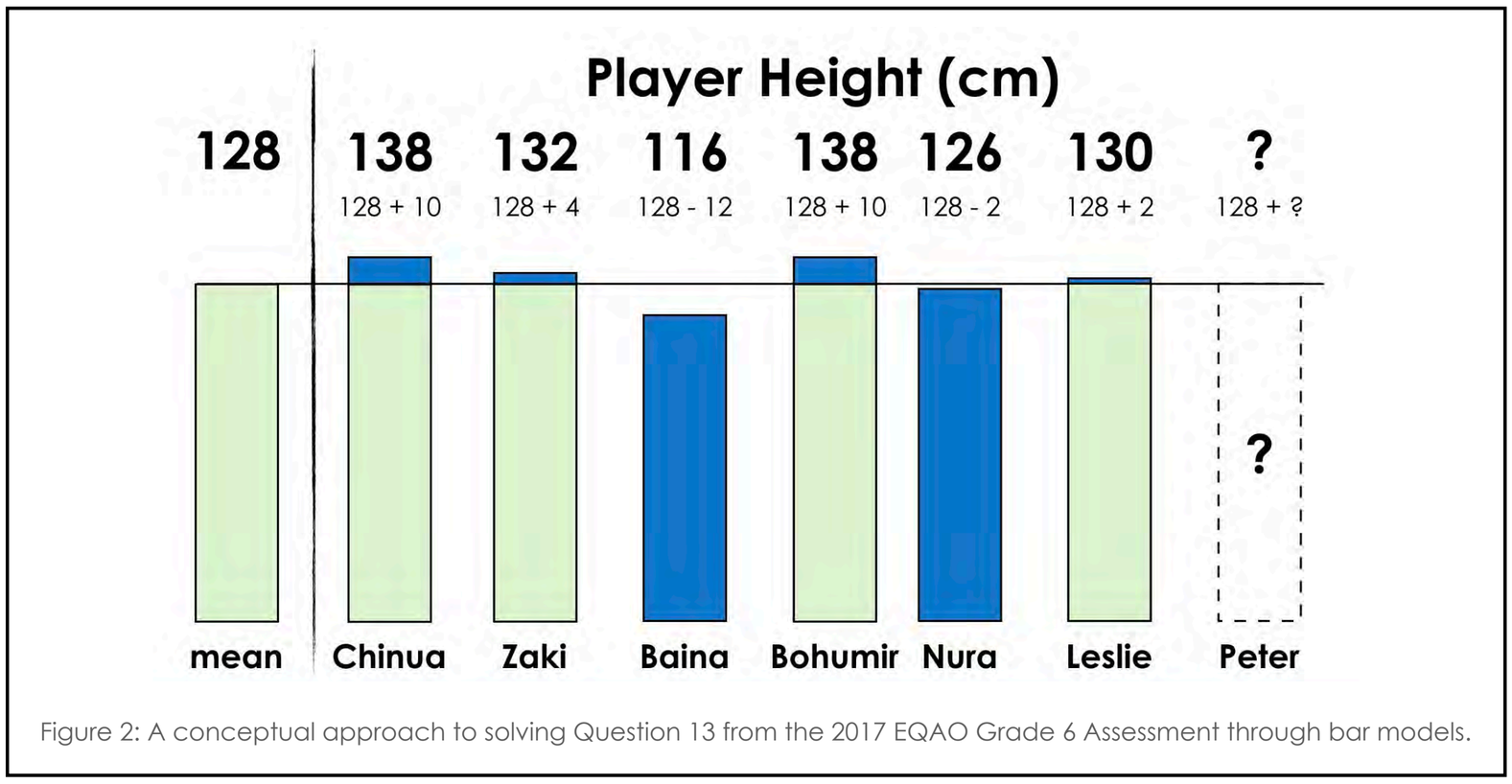
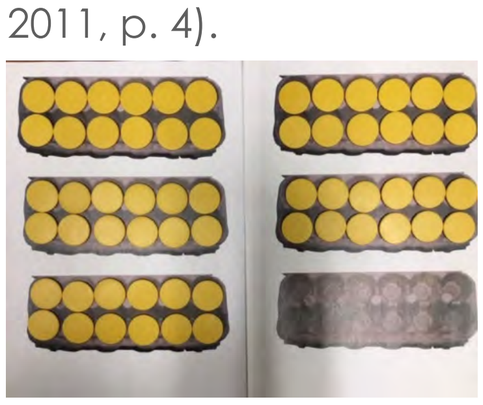


Figure 2: A conceptual approach to solving Question 13 from the 2017 EQAO Grade 6 Assessment through bar models.

total. In reality, these grade 2 students divided 60 by 12 through quotative division.

Students could not articulate that the operator for this problem was division, but they all successfully solved a real world problem through logic and intuition, supported by their prior knowledge and a visual or concrete representation. The role of the educator is to help students gradually make connections between their visual representation and the more abstract notation based on their developmental readiness. "Knowledgeable educators help students transform their everyday mathematics into a more formalized understanding that can be transferred and applied to other situations. Several researchers refer to this as "mathematization" which requires students to abstract, represent and elaborate on informal experiences and create models of their everyday activities (Clements & Sarama, 2009, p. 244). The educator can play an integral role by making meaningful connections between the mathematical strands, the real world and other disciplines, and most importantly, "between the intuitive informal mathematics that students have learned through their own experiences and the mathematics they are learning in school" (Ontario Ministry of Education, 2011, p. 4).



The Impact

One intended outcome of starting concrete is accessibility. The mathematics classroom of the past typically had students who fell into two categories: students who were good at math and students who were not. The use of concrete and visual representations is a way to create equity in the math classroom, and provides an opportunity that supports the achievement of high levels of mathematics for all students. "When we open mathematics, we broaden the number and range of student who engage and do well. This is not an artificial broadening or dumbing down of mathematics; rather, it is a broadening that brings school mathematics closer to real mathematics and the mathematics of the world" (Boaler, 2016).

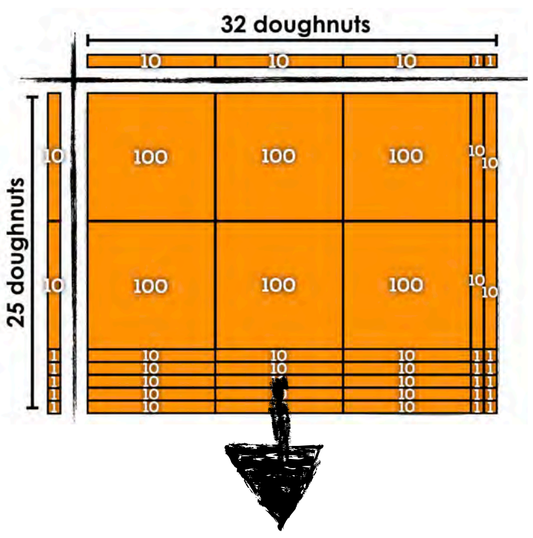
"Students who would normally struggle have hope, as they use concrete materials as tools to develop their conceptual understanding of mathematics. They are not overwhelmed by the numbers in the math or what to do with those numbers. Instead, they can conceptualize, manipulate and develop a concrete understanding of the mathematical concepts with which they are engaging.

The concrete material becomes the bridge for students that helps them anchor their understanding to something tangible, that they have experienced personally, and gives them the confidence to take gradual steps over to the other side, to the realm of more abstract concepts and thinking" (GECDSB Teacher).

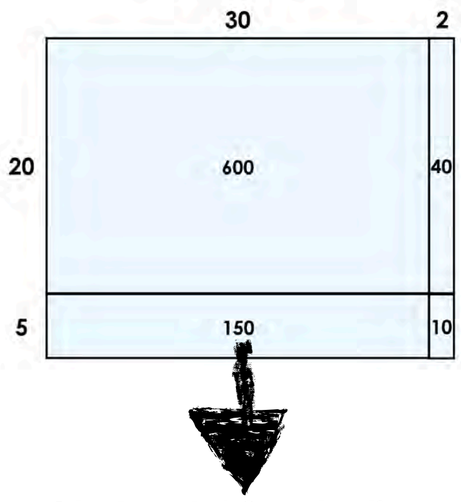
How Many Doughnuts Are There?

Concrete
Actual Doughnuts

Concrete Manipulatives:
Base 10 Blocks



Pictorial / Visual
Area Model: Partial Products



Abstract / Symbolic
Conceptual Algorithm

32
x 25
600
40
150
10
800

MATH TASK FORCE 2.0

Another perceived impact of starting concrete is the influence on student disposition towards mathematics. Students who formerly felt left out of the conversation or left behind can gain some footing, and in turn, a positive mindset around their mathematical ability. The use of concrete materials helps educators lower the floor on every single task. With the GECDSB commitment to the use of manipulatives, the stigma that manipulatives are designated for "those who can't" has begun to dissipate. "I used to feel really nervous about math. I didn't really understand most things and I felt like everyone else knew what to do and I didn't. Now that I know that I can use tools, and my friends use the tools too, I can figure it out. Math just makes so much more sense to me" (GECDSB student).

With an ongoing commitment to promoting the development of the Five Mathematical Proficiencies, identified by the National Research Council in Adding It Up – Helping Children Learn Mathematics, making the mathematics concrete is an approach to support the development of conceptual understanding for both educators and students. Educators were not taught math conceptually.

A focus on concrete and visual representations, allows, but also forces teachers to develop their own conceptual understanding of concepts in order to support student learning. "Teachers need to be able to reason through and justify why certain procedures and properties hold true, to talk about how mathematical language is used, to see the connections between mathematical ideas and to understand how they build upon one another" (Ontario Ministry of Education, 2011, p. 3). It is imperative that the mathematics can be "seen" to promote sense making and to reveal connections. In order for the math to be seen, teachers and students must determine how to model it. Through this process, all participants will engage in critical thinking, problem solving, and patterns and generalizations will be revealed, rather than told. "Each time one prematurely teaches a child something he could have discovered himself, that child is kept from inventing it and consequently from understanding it completely" (Fosnot, 2016).

Using the standard algorithm for multiplication in the Junior Division leaves some students behind and teachers frustrated. Teachers are left to wonder why student cannot remember the steps and why they do not know their facts. Student multiplying double digits forget to add the zero in the second row, and their answer is sometimes completely unreasonable. Teachers then ask the question, "How do students not see that that number doesn't make sense?" Students likely cannot see that the answer is unreasonable because the algorithm means very little to them. At GECDSB, professional development has been dedicated to understanding models and contexts that promote conceptual understanding of multiplication and the development of fact fluency.

Conclusion

The purpose of "making math concrete" is a gateway for creating equity through the development of conceptual understanding. It is an opportunity to make connections, encourage sense making and collaboration. The mathematics of the past was limiting in its ability to promote critical thinking and problem solving. It was reserved for the elite, the "gifted" and the good memorizers. But when the math can be seen, a greater opportunity is possible for all students to love mathematics and to see themselves as mathematicians. "The math I remember in school was rigid. It wasn't about thinking, just rule following. But now I see, math is creative. In reality, it is beautiful and poetic. Math reminds me a lot of music. My school experience didn't give me a chance to see it" (GECDSB Principal).

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