GREATER ESSEX COUNTY DISTRICT SCHOOL BOARD

Rote versus Discovery: Moving the Discussion Forward

The conversation of mathematics proficiency cannot be framed in the context of rote versus discovery learning because neither defines nor describes mathematical proficiency. The GECDSB believes in an integrated approach to teaching and learning that is responsive to the individual needs of the learners and is rooted in a conversation about mathematical proficiency.

The Greater Essex County District School Board provides mathematics education that engages and empowers students through collaboration, communication, inquiry, critical thinking, and problem-solving, to support each student's learning and nurture a positive attitude towards mathematics.

GECDSB, A Vision for Mathematics, 2016

The purpose of these learning briefs is to share the research, discussion and insight garnered from the intensive work of the Greater Essex County District School Board's Math Task Force. These papers are rooted in the GECDSB core beliefs, the Full-Day Early Learning—Kindergarten program and the Ontario Mathematics Curricula for grades 1-8, 9-10, and 11 & 12. The briefs are meant to elevate, enrich and extend the discourse of mathematics education with the intention of encouraging a positive and productive disposition toward mathematics for all learners.

Each paper provides a list of sources to extend the professional conversation and enhance the learning. In addition, live links appear at the end of the papers with connections to various resources.

ROTE VERSUS DISCOVERY: MOVING THE CONVERSATION FORWARD

The rote versus discovery debate has occupied public discussion for years. On either side of the argument stands a passionate group with extensive research that claims to support their position. Each group believes that mathematics is important. Each group wants what is best for students. Interestingly, what divides them is a common understanding of what it means to "do and be good at math." It is this shared definition of mathematical proficiency that ultimately bridges the divide.

Daniel Ansari, of the University of Western Ontario, is professor of psychology and Canadian Research Chair of Developmental Cognitive Neuroscience. His work uses behavioural research methods and neuro-imaging to build an understanding of how children learn about numbers. Ansari (2015) recently published a compelling article with the Canadian Education Association which called for a truce to the "math wars". Ansari (2015) drew attention to the "false dichotomy" that is the math wars, stating that "these two approaches are frequently painted as being completely distinct and diametrically opposed to one another, creating the perception that there is a need to side with one particular view of best practice in math education".

Within this conversation is an array of terminology; words and phrases that unless clearly defined, lead us to talk in circles. Our dialogue must begin by operationalizing the terms being used, or the discourse becomes futile. Ansari describes *rote learning* as being synonymous with the rehearsing or drilling arithmetic facts and *discovery learning* as incorporating the underlying principles of mathematics through hands-on activities and open-problem solving. In the scope of mathematics education we can see how both of these narrow views fall desperately short of defining mathematics proficiency. Neither gives us a start or end point. The conversation of mathematics proficiency cannot be framed in the context of rote versus discovery learning because neither defines nor describes proficiency. Through the article, Ansari repositions the conversation as one of procedural and conceptual knowledge and argues that both are important parts of mathematics. He also calls for education stakeholders to abandon these emotionally-charged debates and use evidence to inform their dialogue.

The Greater Essex County District School Board includes procedural fluency and conceptual understanding as part of its vision for mathematics, but extends the definition based on the broad research of the National Research Council publication, *Adding It Up* (2001). Proficiency in mathematics is defined as: procedural fluency, conceptual understanding, strategic competence, adaptive reasoning, and productive disposition. Based on this definition, our work becomes designing instruction that mobilizes a range of strategies in order to move students toward proficiency (National Research Council, 2001).

Mathematics Proficiency

I want my child to know her times tables. *Absolutely*. I want my son to understand the concept of number. *Of course*. I want my students to solve problems using multiple strategies. *Definitely*. I want my daughter to love and excel in math. *Certainly*. I want my students to think mathematically and be able to justify their thinking. *Yes, without a doubt!*

Proficiency in Mathematics cannot be defined by any one facet, application, strategy, or attitude. It is an interweaving of five competencies, each distinct but with no one strand encompassing the entirety (National Research Council, 2001). It is the entwining of the threads that becomes the framework for mathematics proficiency and this is grounded in the goals and expectations of the Ontario Curriculum grades 1-8: Mathematics (2005). The proficiencies have been described in great detail in the publication *Adding It Up,* where the authors boldly state:

> *The most important observation we make here, one stressed throughout this report, is that the five stands are interwoven and interdependent in the development of proficiency. Mathematical proficiency is not a one dimensional trait, and it cannot be achieved by focusing on just one or two of the strands. (National Research Council, 2001).*

The Five Threads of Proficiency

Skemp (1976) argued that it is not enough for students to understand *how* to perform various mathematical tasks; they must understand *why*. He used the term "relational understanding" and explained that it is an appreciation of the underpinnings, ideas and relationships in mathematics. The first of the threads of

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MATH TASK FORCE

proficiency is conceptual understanding, which is the *why* of math. It is the ability to understand mathematical concepts, operations, and relationships, and the contexts in which they are useful. For example, when considering a multiplication question such as 55x24, a person with conceptual understanding can see that the problem could be represented as repeated addition, or as the area of a quadrilateral, the number of seats in a theatre, and any other scenario they can conceive.

Students with conceptual understanding are able to arrange representations in a variety of ways and use these representations to build new ideas. They can discuss the similarities or differences among these representations and make connections between "clusters" of mathematical principles, laws and properties (National Research Council, 2001, p. 120).

Building on this idea is the second thread of procedural fluency. This is the skill of carrying out procedures flexibly, accurately, and efficiently, and understanding the context in which the procedures should be applied. In the example of 55x24, a person with procedural fluency may apply a known method such as organizing the numbers horizontally and carrying out a standard algorithm. Being able to estimate and complete mental computations is also an important part of procedural fluency. Students need to be efficient and accurate in performing basic computations and a good conceptual understanding helps to support procedural fluency.

In school mathematics, procedural fluency and conceptual understanding are sometimes positioned as opposing concepts. This could not be further from the truth. The authors of *Adding it Up* clarify:

> *Procedural fluency and conceptual understanding are often seen as competing for attention in school mathematics. But pitting skill against understanding creates a false dichotomy. As noted earlier, these two are interwoven. Understanding makes learning skills easier, less susceptible to common errors and less prone to forgetting. By the same token, a certain level of skill is required to learn many mathematical concepts with understanding, and using procedures can help strengthen and develop that understanding* (National Research Council, 2001, p. 122).

Being able to solve mathematical problems is a large part of what it means to be proficient in mathematics. The third thread of proficiency is strategic competence, which is the ability to formulate, represent and solve mathematical problems using effective strategies. Devising a strategy includes being able to manipulate the process of problem-solving by formulating and selecting approaches. Students with strategic competence will exhibit conceptual understanding when they select and organize their solution, and procedural fluency when they carry out their strategy with efficiency. Strategic competence is an integral part of procedural fluency because over time and with experience, students see the value of selectiveness and efficiency of procedures. For example, consider when it is useful to multiply instead of adding repeatedly. Students need to be able to "replace by more concise and efficient procedures, those cumbersome procedures that might at first have been helpful in understanding the operation" (National Research Council, 2001. p. 126).

The fourth thread of proficiency, adaptive reasoning, is the capacity for logical thought, reflection, explanation, and justification. It is not enough to just select and carry out a strategy. Deductive reasoning is used to make conclusions using facts, definitions, rules, or properties. Mathematics learning develops when people are able to articulate the proofs and mathematical decisions they made, including: why a certain strategy was selected, why it was the most effective, and how they know they were successful or not. With the assistance of representations, even young children can demonstrate their justifications and reasoning. It is important to consider that, "it is not sufficient to justify a procedure just once… Students need to use new concepts and procedures for some time and to explain and justify them by relating them to concepts and procedures they already understand" (National Research Council, 2001, p. 130).

There has been significant work done in the area of Mathematical Mindsets by leaders like Jo Boaler (2015), who explain how our beliefs are strongly tied to our behaviour. Thus, seeing mathematics as useful and worthwhile helps to empower children to engage deeply in their learning. The fifth thread, productive disposition, is an inclination to see mathematics as beneficial and valuable. It allows students to see where and how mathematics can be applied, not only to the world around them, but in service of the intrinsic beauty of the discipline. Productive disposition is a tenacious belief that mathematics is not arbitrary or irrelevant, but understandable and worth the effort.

Developing a productive disposition does not

Conceptual understanding is the ability to understand mathematical concepts, operations, and relationships, and the contexts in which they are useful.

Procedural fluency is the skill of carrying out procedures flexibly, accurately, and efficiently, and understanding the context in which the procedures should be applied.

Strategic competence is the ability to formulate, represent and solve mathematical problems using effective strategies.

Adaptive reasoning is the capacity for logical thought, reflection, explanation, and justification.

Productive disposition is an inclination to see mathematics as beneficial and valuable. It allows students to see where and how mathematics can be applied, not only to the world around them, but in service of the intrinsic beauty of the discipline.

Proficiency develops over time, with practice, instruction, feedback, support and opportunity. As educators we take up the challenge of synchronically developing each of the strands of proficiency from kindergarten through secondary school.

Our students need to learn mathematics, and they need mathematics to learn.

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MATH TASK FORCE

mean that we eliminate obstacles and challenges. Instead, we capitalize on the other strands of proficiency and provide opportunities for students to make mathematics meaningful through their perseverance and enthusiasm.

Proficiency develops over time, with practice, instruction, feedback, support, and through opportunity. As educators we take up the challenge of concurrently developing each of the strands of proficiency from kindergarten through secondary school. Becoming proficient in mathematics *is the start and end point*. For too long we have rested on incomplete definitions of the purpose of school mathematics and have engaged in misleading and distracting quarrels.

Our Full-Day Early Learning—Kindergarten program and the Ontario Mathematics Curricula for grades 1-12 both identify and promote proficiency. They provide the anchor and direction for mathematics instruction in Ontario. The expectations identify the classroom actions and the interconnectedness of the threads. *Explore, represent, design, justify, solve, compare* — these verbs direct the actions of proficiency. Our curriculum clearly identifies what proficiency looks like in a classroom. The work of educators is to design mathematics instruction that builds the strength of each thread in order to weave a rich and robust tapestry of proficiency.

Our students need to learn mathematics, and they need mathematics to learn. In order to elevate the discourse of mathematics education, our conversations must be rooted in proficiency because it is this aim toward excellence which will facilitate students to excel in their applications of mathematics and position them to realize its boundlessness.

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LINKS

Dr. Chris Suurtamm—Planning Moves for Teachers (https://vimeo.com/136750780)

Dr. Cathy Fosnot—Basic Fact or Conceptual Understanding : A False Dichotomy (https://vimeo.com/104110510)

Dr. Cathy Fosnot—Conceptual Understanding and Procedural Fluency: We Need Both (https://vimeo.com/137299162)