GREATER ESSEX COUNTY DISTRICT SCHOOL BOARD

MATH TASK FORCE REPORT 2016

April 5, 2016

ACKNOWLEDGEMENTS

This report is the culminating effort of a team of extremely talented and dedicated individuals. This work would have not have been possible without the commitment, persistence, expertise and exceptional passion of the Math Task Force Committee. Each member has brought a unique perspective to this process and their collective contributions and efforts have forged a path for the teaching and learning of mathematics within the Greater Essex County District School Board.

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GECDSB Math Task Force
Project Overview

In the fall 2014, the Greater Essex County District School Board released *A Vision for Mathematics*, which set the aims of mathematics education for the GECDSB. This document was developed from an extensive review of research, and grounded in the principles of the Full-Day Early Learning—Kindergarten program and the Ontario Mathematics Curricula for grades 1-8, 9-10, and 11 & 12. It was instrumental in extending the discourse of mathematics education toward a comprehensive definition of mathematical proficiency.

> ***"The GECDSB provides mathematics education that engages and empowers students through collaboration, communication, inquiry, critical thinking and problem-solving, to support each student's learning and nurture a positive attitude towards mathematics."***
> GECDSB: A Vision for Mathematics

Conversations about mathematics teaching and learning continued, and greater attention was placed on how to best support student achievement in mathematics. In the spring of 2015, the Trustees of the Greater Essex County District School Board approved a motion for the formation of a "Math Task Force". The purpose of this Task Force was to examine the practices of mathematics teaching and learning from multiple perspectives within the GECDSB, and to advise Senior Administration and the Board of Trustees as to how best to support future planning in the area of mathematical teaching and learning.

The GECDSB Math Task Force is comprised of a diverse group of individuals including Trustees, classroom educators, school administrators, parent representatives, central office staff, university and community experts, and university students. In addition to the committee members, external experts from the field of education provided input as "critical friends" of the cooperative work.

The combined work of the GECDSB Math Task Force engaged in extensive dialogue and a comprehensive review of relevant research. Feedback was received from stakeholders, which informed the development of considerations. The following document is a summary of the collective work, as well as the considerations for review and implementation. "The GECDSB provides mathematics education that engages and empowers students through collaboration, communication, inquiry, critical thinking and problem-solving, to support each student's learning and nurture a positive attitude towards mathematics."

A VISION FOR MATHEMATICS

"The GECDSB provides mathematics education that engages and empowers students through collaboration, communication, inquiry, critical thinking and problem-solving, to support each student's learning and nurture a positive attitude towards mathematics."

Whenever we strive to improve in any way, a vision of what that improvement might be is essential. We need to know what we are working towards and why that is important. We also need to know how we plan to reach that vision.

This vision has been developed specifically by and for the Greater Essex County District School Board through consultations with a wide variety of stakeholders including elementary and secondary teachers and administrators, program staff, Student Success, and Special Education departments. The intent of this vision, and the related strategies and approaches to mathematics teaching and learning, is to support schools and educators as they reflect on the needs of their students and how they will address them as part of their ongoing School Improvement Plans.

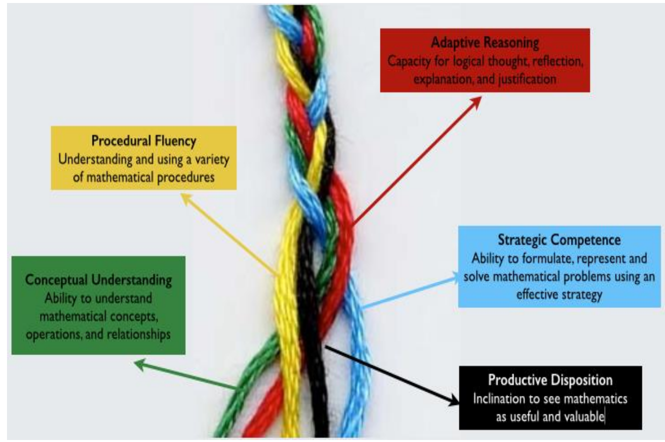
Within this vision there are various responsibilities we assume. As a school board, we believe our responsibilities are to create conditions for mathematics learning:

- where competent and knowledgeable educators integrate instruction and assessment;
- where educators and administrators are committed to ongoing learning about mathematics and mathematics instruction;
- where learning environments nurture positive attitudes towards mathematics; and
- where all students have opportunities and support to learn significant mathematics with depth and understanding.

It is the belief of the board that where this vision is actively pursued, and where these responsibilities are met, students' achievement in mathematics will increase.

A Picture of Mathematical Literacy

What is Mathematical Literacy?



Conceptual Understanding is the ability to understand mathematical concepts, operations, and relationships.

Procedural Fluency is the skill in carrying out procedures flexibly, accurately, and efficiently, and knowing when the procedures should be applied.

Adaptive Reasoning is the capacity for logical thought, reflection, explanation, and justification.

Strategic Competence is the ability to formulate, represent and solve mathematical problems using an effective strategy.

Productive Disposition is the inclination to see mathematics as useful and valuable.
In order to begin any conversation around improving mathematics we need to share a common understanding of mathematical literacy.

For the complete version of the GECDSB: A Vision for Mathematics (See Appendix A).

LITERATURE REVIEW

The following is a review of the research related to various topics investigated by the Greater Essex County District School Board – Math Task Force. The purpose of this literature review is to inform the formulation of considerations. The review is organized according to the scope of each Ad-Hoc Committee and the general topics investigated by each group.

UNDERSTANDING RECIPROCAL PARTNERSHIPS

The Reciprocal Partnerships Committee investigated the current and prospective practices, relationships and partnerships that support mathematics teaching and learning. There were several topics of research that had a significant impact on the work and considerations of this group.

The Importance of Family Engagement

Parental involvement in their children's learning is widely acknowledged in research as having a positive impact on student achievement. More importantly, parental engagement has been identified as a mitigating difference in socioeconomic status and student achievement (Fan & Chen, 2001; Henderson & Mapp, 2001; Bolivar & Chrispeels, 2011; Ma, She & Krenn, 2013). The difference between the terms *involvement* and *engagement* is slight, but significant. The dictionary defines *involve* as "to have or include," whereas the meaning of the term *engage* is "to mesh into a working arrangement." (Merriam-Webster, 2014). Thus, parental involvement implies *doing to*; where parental engagement, implies *doing with*. Although the literature typically uses the term *parent*, for the purposes of this paper the term family is used to as to encourage inclusiveness of all contexts. Family engagement can and does make a positive difference for students.

In a report commissioned by the Council of Directors of Education and the Institute of Education Leadership entitled *Strong Districts and Their Leadership,* Kenneth Leithwood (2013) identifies a productive relationship with staff and other stakeholders as one of the critical features of strong school districts. The research goes on to describe how within these contexts, there is an emphasis on joint responsibility for working toward success of the common goal of improved student achievement (Leithwood, 2013). In addition, the district imperatively builds relationships with the community in order to accomplish their mission and goal. This connects to the primary and critical feature of strong districts, which is a broadly shared mission, vision and goals founded on ambitious images of the educated person.

Specific to the notion of family engagement and support for mathematics, Vukovic, Roberts, & Wright (2013) state that policies and programs targeting involvement in mathematics should focus on home-based practices that do not necessarily require technical mathematical skills. Further noted is the idea that parents and families should receive training, resources and support on culturally appropriate ways to create home learning environments that foster high expectations for their children's success in mathematics.

This research holds true for elementary students, but is also applicable to secondary students. Jeynes (2007) specifically focused on the achievement of secondary school students. One of the patterns that emerged from the findings was that subtle aspects of parental involvement, such as parental style and expectations, had a greater impact on student achievement than more demonstrative factors like household rules and parent participation in school functions.

Home and School Communication

Communication between families and schools is crucial. It is an essential condition in establishing authentic and productive parent/guardian school partnerships, but it is important to note that one-way communication has significant limitations. There are several characteristics of high-quality home-school communication which are identified in the Ontario Ministry of Education Capacity Building Series - Parent Engagement (2012). First, *trust* is vital in developing a productive relationship with parents/guardians. A foundation of trust should promote dialogue and move toward increased reciprocity in order to facilitate awareness and action. Another characteristic of effective engagement is *awareness,* which is necessary when identifying the needs and challenges that students are facing. Without it, the student's underlying challenges may not surface, making it difficult to develop and target differentiated support (Capacity Building – Parent Engagement Oct. 2012).

It is critical to recognize that there is no single form of communication or support that surpasses all others. Schools must engage with families through means that are relevant, appropriate and fitting to the unique contexts of their communities. As well, schools need to continuously investigate and extend new opportunities for engagement as both communication methods and family needs are continuously changing. As Ferlazzo notes, "A school striving for parent engagement…tends to lead with its ears,listening to what parents think, dream and worry about. The goal of family engagement is not to serve clients but to gain partners" (2011, p. 10).

As we build greater understanding of the impact of reciprocity and relationships between schools and the community, researchers like Kruse and Louis identify that 'deep-seated' changes in the culture of schools are unlikely to occur without action to create more fundamental bonds with the community" (p.7). As well, the research of Leithwood (2015) uncovers that schools embarking on family engagement efforts targeted at improving student outcomes should build in sufficient time and opportunity for staff learning. This research affirms efforts to train staff and develop comprehensive and targeted plans to improve family engagement.

Building Partnerships – Locally and Globally

Partnerships between schools and families are a cornerstone of student success. However, alternative partnerships, like those with local and global partners, are also important in leveraging and mobilizing knowledge to improve student outcomes. Mathematics achievement is a global focus, and information shared among various groups serves to enhance the learning of the collective. One example of global partnership is that of the Reciprocal Learning Program. This is an international initiative developed between the University of Windsor, the Greater Essex County District School Board, Southwest University, and Chong Qing schools, which has become the essential part of a 7-year Canada-China Reciprocal Learning Partnership Project funded by the Social Sciences and Humanities Research Council of Canada (SSHRC) with five other Canadian and Chinese partner institutions (Xu and Connelly, 2013).

Partnerships such as these serve to enhance learning for all participating parties because it is through dialogue and discourse that educators are able to build a better understanding of practices that support student learning. These cross-cultural perspectives support new approaches to research on curriculum and their application in classrooms. The goals of the Reciprocal Learning Program are: to provide an exceptional learning experience for teacher candidates, educators and administrators; to expand perspectives regarding societies of increased diversity; to foster international collaboration among faculty members who are interested in cross-cultural studies; to promote multicultural education; and to enhance international education relationships (Xu, 2011).

Schools and school-systems benefit from rich, responsive and reciprocal relationships with local and global partners. Practices that support structures, systems and professional learning tied to the engagement of these partners will serve to enhance the experience of all stakeholders and ultimately benefit students.

UNDERSTANDING SYSTEM PRACTICES

The System Practices Ad-hoc Committee examined current system practices and research of effective systems in order to support mathematics teaching and learning. There were a number of areas of research that had significant impact upon the work and considerations of this group.

Foundational Principles for Improvement in Mathematics

In September 2010, the Ontario Ministry of Education brought together a Mathematics Teaching and Learning Working Group in order to "identify what it would take to bring greater depth and coherence to the K–12 mathematics program" (p. 1). This collaborative group included various experts from diverse backgrounds who came together to guide and advise the Assistant Deputy Ministry on means of improving mathematics achievement in Ontario. The work of the collective is an important grounding for the work of the Greater Essex County District School Board Math Task Force System Practices Committee, as the research done by this group is *reflective of* and *reflected in* the work of the "Seven Foundational Principles for Improvement in Mathematics, K–12."

The document identifies seven key principles which guide school and system practices in the area of mathematics. They are:

- Focus on mathematics.
- Coordinate and strengthen mathematics leadership.
- Build understanding of effective mathematics instruction.
- Support collaborative professional learning in mathematics.
- Design a responsive mathematics learning environment.
- Provide assessment and evaluation in mathematics that supports student learning.
- Facilitate access to mathematics learning resources.

(Paying Attention to Mathematics Education K-12, 2011)

Each of these principles is imperative in guiding effective school and system practices. They serve as a comprehensive framework for system and school planning of mathematics teaching and learning.

Defining Mathematical Proficiency

Proficiency of mathematics must be both a start and end point for any discourse in mathematics education. Recently, Daniel Ansari (CEA, 2015), of the University of Western Ontario, published a compelling article with the Canadian Education Association which called for a truce to the "math wars." Ansari drew attention to the "false dichotomy" of rote and discovery models of learning, stating that "these two approaches are frequently painted as being completely distinct and diametrically opposed to one another, creating the perception that there is a need to side with one particular view of best practice in math education" (2015). Through the article, Ansari repositions the conversation as one of procedural and conceptual knowledge and argues that both are important parts of mathematics. He also calls for education stakeholders to abandon these emotionally-charged debates and use evidence to inform their dialogue (Ansari, 2015).

This vision of mathematical proficiency is extended by the broad research of the National Research Council publication, *Adding It Up* (2001). According to their research, proficiency in mathematics is defined as: procedural fluency, conceptual understanding, strategic competence, adaptive reasoning, and productive disposition. Based on this definition, our work becomes designing instruction that mobilizes a range of strategies in order to move students toward proficiency (National Research Council, 2001).

Proficiency in Mathematics cannot be defined by any one facet, application, strategy or attitude. It is an interweaving of five competencies: each distinct, but no one strand encompassing the entirety (National Research Council, 2001). It is the entwining of these threads that becomes the framework for mathematics proficiency, and this is grounded in the goals and expectations that are rooted in the GECDSB core beliefs, the Full-Day Early Learning—Kindergarten program and the Ontario Mathematics Curricula for grades 1-8, 9-10, and 11 and 12. The proficiencies have been described in great detail in the publication, *Adding It Up,* where the authors boldly state:

> *The most important observation we make here, one stressed throughout this report, is that the five stands are interwoven and interdependent in the development of proficiency. Mathematical proficiency is not a one dimensional trait, and it cannot be achieved by focusing on just one or two of the strands. (National Research Council, 2001)*

Understanding the Five Threads of Mathematics Proficiency

Skemp (1976) argued that it is not enough for students to understand *how* to perform various mathematical tasks - they must understand *why.* He used the term "relational understanding" and explained that it is an appreciation of the underpinnings, ideas and relationships in mathematics. The first of the threads of proficiency is *conceptual understanding*, which is the *why* of math. It is the ability to understand mathematical concepts, operations, and relationships and the contexts in which they are useful. Students with conceptual understanding are able to arrange representations in a variety of ways and use these representations to build new ideas. They can discuss the similarities or differences among these representations and make connections between "clusters" of mathematical principles, laws and properties (National Research Council, 2001, p. 120).

Building on this idea is the second thread of mathematical proficiency, *procedural fluency*. This is the skill of carrying out procedures flexibly, accurately, and efficiently, and understanding the context in which the procedures should be applied. Being able to estimate and complete mental computations is an important part of procedural fluency. Students need to be efficient and accurate in performing basic computations, and a deep conceptual understanding helps to support procedural fluency.

In school mathematics, procedural fluency and conceptual understanding are sometimes positioned as opposing concepts. This could not be further from the truth. The authors of *Adding it Up* clarify:

> *Procedural fluency and conceptual understanding are often seen as competing for attention in school mathematics. But pitting skill against understanding creates a false dichotomy. As noted earlier, these two are interwoven. Understanding makes learning skills easier, less susceptible to common errors and less prone to forgetting. By the same token, a certain level of skill is required to learn many mathematical concepts with understanding and using procedures can help strengthen and develop that understanding. (National Research Council, 2001, p. 122)*

The third thread of proficiency is *strategic competence*, which is the ability to formulate, represent and solve mathematical problems using effective strategies. Devising a strategy includes being able to manipulate the process of problem-solving by formulating and selecting approaches. Students with deep strategic competence will develop conceptual understanding when they select and organize their solution, and procedural fluency when they carry out their strategy with efficiency. Strategic competence is an integral part of procedural fluency because over time and with experience, students see the value of selectiveness and efficiency of procedures. For example, consider when it is useful to multiply instead of adding repeatedly. Students need to be able to "replace by more concise and efficient procedures, those cumbersome procedures that might at first have been helpful in understanding the operation" (National Research Council, 2001. p. 126).

The fourth thread of proficiency, *adaptive reasoning*, is the capacity for logical thought, reflection, explanation, and justification. It is not enough to just select and carry out a strategy. Deductive reasoning is used to make conclusions using facts, definitions, rules, or properties. Mathematics learning develops when people are able to articulate the proofs and mathematical decisions they made, including: why a certain strategy was selected, why it was the most effective, and how they know if they were successful or not. With the assistance of representations, even young children can demonstrate their justifications and reasoning. It is important to consider that "it is not sufficient to justify a procedure just once ... Students need to use new concepts and procedures for some time and to explain and justify them by relating them to concepts and procedures they already understand" (National Research Council, 2001, p. 130).

The final thread of proficiency underpins all other areas and is termed *productive disposition*. This is defined as the inclination to see mathematics as useful and viable. There has been significant work done in the area of Mathematical Mindsets by leaders like Jo Boaler (2015), who explain how our beliefs are strongly tied to our behaviour. Thus, seeing mathematics as useful and worthwhile helps to empower children to engage deeply in their learning. It allows students to see where and how mathematics can be applied, not only to the world around them, but in service of the intrinsic beauty of the discipline. Productive disposition is a tenacious belief that mathematics is not arbitrary or irrelevant, but understandable and worth the effort.

Proficiency develops over time, with practice, instruction, feedback, support, and through opportunity. As educators, we take up the challenge of concurrently developing each of the strands of proficiency from kindergarten through secondary school. Becoming proficient in mathematics is both the start and end point. For too long we have rested on incomplete definitions of the purpose of school mathematics and have engaged in misleading and distracting quarrels.

Building Expertise in Mathematics Teaching

The National Council of Teachers of Mathematics (NCTM) has cited that teacher knowledge, understanding and skill are of central importance in the teaching of mathematics (2000). It is therefore no surprise that the role of content expertise is a critical debate among educators, parents and policy makers. In order to become successful citizens of the 21st century, it is vital that we educate children to become proficient in mathematics, and our success is in large part determined by effective teaching. The concept of expertise in the teaching of mathematics requires careful study, and a review of the research raises many significant considerations that need to be closely examined at classroom, school and system levels.

In order to better understand the crux of the issue, we need to explore the concepts of experience, experts, and expertise. According to John Hattie, experienced teachers are those who have years of practice and familiarity teaching, however Hattie identifies that "experience" and "experts" are not necessarily one in the same. He states:

> *Experts and experienced teachers do not differ in the amount of knowledge they have about curriculum matters or knowledge about teaching strategies. But experts do differ in how they organize and use this content knowledge.* (Hattie, 2003)

Certainly within the content rich area of mathematics there is a desire to identify and leverage our expert educators. Conversations continue about math experts and their roles in schools. In order to best serve the interests of our students, we need to expand our definition of *expert* to include *expertise*. If an expert is solely defined as someone who has formal education in a content area, then we are missing crucial aspects of teaching expertise.

The well-known work of Lee Shulman (1987) on pedagogical-content-knowledge (PCK) has long demonstrated that teaching expertise extends beyond content area knowledge. Shulman describes content as the "what" and pedagogy as the "how" of teaching. According to his work, pedagogical-content-knowledge is a highly specialized skill-set that differs from subject specific knowledge and general pedagogical knowledge (Shulman, 1987). Within the context of mathematics teaching we need to adopt a comprehensive definition of the term "expert" which is grounded in research and inclusive of all the domains of expertise. As Ball, Hill and Bass (2005) share, the knowledge needed to do mathematics is different from the knowledge needed to teach mathematics.

The use of specialty mathematics educators is an accepted practice in secondary schools and in some elementary schools who adopt a "rotary" model. This model can have advantages when specialty teachers have demonstrated expertise, but the limitations of this model must be critically considered. The drawbacks include logistical implications for staffing and scheduling as well as the isolation of mathematics instruction to specific periods. This structure impairs the opportunity for meaningful integration of mathematics throughout the instructional day, and this integration is a central principle of the Full-Day Early Learning—Kindergarten program and the Ontario Mathematics Curricula for grades 1-8, 9-10, and 11 & 12. In addition, this model restricts professional learning to only specialty teachers, creating significant long-term impacts on schools and a school-system (Gerretson, Bosnick and Schofield, 2008).

Students engage in mathematics learning throughout their years of elementary and secondary school. Effectively supporting this learning requires all educators to continue to develop and refine their expertise in mathematics. It is critical for our schools and school system to support a deeper and broader understanding of mathematics teaching and learning for all educators.

Building Expertise in Mathematics Leadership

The ability of the school-based administrator to assume the role of leadership for school improvement is a significant determining factor in the success of any educational advancement (Leithwood, 1992). The *Seven Foundational Principles for Improvement in Mathematics, K–12*, identifies that "All system, board and school leaders commit to providing the resources that support the most effective teaching and learning of mathematics for all students" (p. 5). This document recognizes the role of all system, board and school leaders as essential in supporting mathematics teaching and learning (2010, p. 5). With this said, the Ontario Ministry of Education Mathematics Teaching and Learning Working Group identifies that there should be very specific and thoughtful planning of leadership capacity specific to mathematics (2010, p.5). These planning factors include:

- focusing board and school improvement plans, based on classroom, school and system level student achievement data, on mathematics;
- aligning resources, including staffing, with mathematics priorities to support students with the greatest mathematics need;
- engaging in open-to-learning conversations, regarding alternative ways to address student needs in mathematics;
- building on professional learning to support teachers in their ongoing effective teaching and learning practices in mathematics;
- participating in professional learning to develop knowledge of what good teaching and learning in mathematics looks like.

(Paying Attention to Mathematics Education, *Seven Foundational Principles for Improvement in Mathematics, K–12*, 2010, p. 5)

School Self-Assessment and District Review
The Ministry of Education *School Effectiveness Framework K-12 (2013)* is a school self-assessment tool. It should serve to:

- help educators identify areas of strength, areas requiring improvement and next steps;
- act as a catalyst for shared instructional leadership through collaborative conversations focused on high levels of student learning and achievement;
- promote inquiry focused on student learning, achievement and well-being that informs goals and effective teaching and learning practices/strategies;
- support educators in determining explicit, intentional and precise improvement planning decisions which inform monitoring and feedback for continuous improvement and future planning in relation to enhanced student learning, achievement and well-being;
- maintain communication with stakeholders to foster increased public confidence about school effectiveness;
- build coherence in and across schools and districts.

The SEF Framework also outlines how school self-assessment functions as a foundation of a District Review Process.

The District Review Process is intended to promote a culture of reflection, collaborative inquiry and shared responsibility for continuous improvement at both the board and the school level. To be successful, the District Review Process must be collaborative, collegial, equitable, inclusive, and generate respectful interactions. It must have open, honest and transparent communication throughout the process, and be consistent with the intent of the *Ontario Leadership Framework, the District Effectiveness Framework* and this document. A non-evaluative, supportive stance is essential. The uniqueness of each school and the strategies undertaken to promote increased student learning, achievement and well-being provide the context for the District Review Process which:

- builds upon the school self-assessment process (School Self-Assessments precede the District Review Process);
- supports schools in the complex inquiry process and ongoing self-reflection, specific to the indicators each school has identified in their school improvement plan;
- helps districts develop a deeper understanding of the strengths and areas of need in elementary and secondary schools;

- supports the notion of continuous improvement for schools and districts;
- informs board improvement planning, goal setting, professional learning supports and budget processes;
- follows a cycle that allows all schools to benefit from a District Review Process within a reasonable timeframe (e.g., three to five years).

The School Team and the District Team work together, using the school's completed self-assessment to determine collaboratively the areas of focus and related evidence to be collected during the school visit. The District Team conducts a school visit to collect data and observations to be analyzed by team members. The principal and educators work together to analyze the evidence for the purpose of sharpening the focus, setting the direction for capacity-building, and developing actions that will be incorporated into the cycle of ongoing school improvement planning and implementation.

Results from the District Review Process are analyzed to determine trends and patterns to inform:

- board improvement planning and capacity-building needs;
- allocation of resources (human, material, financial) to schools for the next school year through the board's budget process;
- expectations and supports for coaching and monitoring.

The *School Effectiveness Framework* (2013) processes are connected to the *Ontario Leadership Framework* (2013). Throughout the province, it has been demonstrated that effective districts deeply engage in these review processes. This framework has been a powerful catalyst for systemic change which support improvements in student achievement.

Pedagogical Systems for Mathematics

The following was shared by the Ministry of Education Student Achievement Division during "System Implementation and Monitoring" and "Numeracy K-12 Capacity Building Sessions", 2015.

Based on Anthony and Walshaw's *Best Evidence Synthesis* (2007), effective mathematical pedagogy is a coherent system rather than a set of discrete, interchangeable strategies. This pedagogical system encompasses: a non-threatening classroom environment, instructional tasks, tools and representations and classroom discourse.

A Non-Threatening Classroom Environment

Anthony and Walshaw (2007) describe a non-threatening classroom environment as one in which there is an acknowledgement that all students, irrespective of age, have the capacity to become powerful mathematics learners. They write about the development of a trusting climate yet also a culture of academic press in which "teachers who truly care about their students have high yet realistic expectations about enhancing students' capacity to think, reason, communicate and reflect upon their own and others' understanding." Supported by school leadership and fostered by strong home/school/community partnerships and communities of practice, teachers work at developing interrelationships that create spaces for learners to develop their mathematical identities and cultural identities.

Instructional Tasks

Anthony and Walshaw (2007) describe Instructional Tasks as focused on the solution of genuine mathematical problems. The most productive tasks and activities are those that allow students to access important mathematical concepts and relationships, to investigate mathematical structure, and to use techniques and notations appropriately. Tasks need to provide the appropriate challenge, provide opportunities for cognitive engagement and understanding. In order to make mathematics more meaningful and accessible for all learners, mathematics curricula frequently advocate the use of contexts – a real or imaginary setting for a mathematical problem, which illustrates the way the mathematics is used.

Tools and Representations
Anthony and Walshaw (2007) describe Tools as thinking spaces that help to organize mathematical thinking. Symbolic artefacts or inscriptions/representations characteristic of mathematics include the number system, algebraic symbolism, graphs, diagrams, models, equations, notations for fractions, functions, calculus, pictorial imageries, analogies, metaphors, models (such as pizzas, chocolate bars and ten frames), examples, stories, illustrations, textbooks, rulers, clocks, calendars, technology (such as computers, calculators, computers, calculators and digital resources, and problem contexts). Anthony and Walshaw (2007) look at how teachers use tools to support students' learning, and how students use tools to reorganize their activity.

Classroom Discourse
Effective teachers engage in classroom discourse by valuing students' ideas, exploring students' answers, incorporating students' background knowledge, and encouraging student explicit communication about mathematical learnings. Anthony and Walshaw (2007) describe Classroom Discourse as more than developing a respectful, trusting and nonthreatening climate for discussion and problem solving – it also includes socializing students into a larger mathematical world that honours standards of reasoning and rules of practice. Articulating comprehensible explanations about mathematical concepts is a learned strategy – and the art of communicating about mathematics has to be modelled through explicit instruction. Mathematical language involves more than vocabulary and technical usage; it encompasses the ways that expert and novice mathematicians use language to explain and justify concepts. Competency in mathematics allows the student to understand discussions about mathematics and comprehend the mathematical way of speaking. Effective classroom discourse involves supporting students in developing the skills of explanation, argumentation and justification.

In order to inform practices and policies for school mathematics, it is imperative that we look beyond our personal experience and critically examine evidence provided by research. Our Ontario Mathematics Curricula ground us in this conversation and states that:

> *An information and technology-based society requires individuals who are able to think critically about complex issues, analyse and adapt to new situations, solve problems of various kinds, and communicate their thinking effectively. The study of mathematics equips students with knowledge, skills, and habits of mind that are essential for successful and rewarding participation in such a society. To learn mathematics in a way that will serve them well throughout their lives, students need classroom experiences that help them develop mathematical understanding; learn important facts, skills, and procedures; develop the ability to apply the processes of mathematics; and acquire a positive attitude towards mathematics. (The Ontario Curriculum Grades 1-8: Mathematics, 2005)*

UNDERSTANDING PROFESSIONAL DEVELOPMENT, CURRICULUM AND RESOURCES

The Professional Development, Curriculum and Resources Ad-hoc Committee explored structures and practices of professional learning, and resources of mathematics teaching and learning. There were several themes of research that had a substantial bearing on the work and considerations of this group.

Educators Need to Learn all the Time

A focus on refining mathematics learning to improve student outcomes has been a priority for Ontario's Ministry of Education in recent years. With the understanding that an educator is the key to a highly effective mathematics program (Bruce and Flynn, 2013), attention must be given to what professional learning educators might need to develop and support their mathematical teaching.

The National Council of the Teachers of Mathematics states that teachers need to understand the big ideas of mathematics and be able to represent mathematics as a coherent and connected enterprise. (NCTM, 2000, p. 17). Further, the National Research Council (2001) links the five proficiencies of mathematical literacy to the educator knowledge and skills required to optimize student learning. These interrelated components include:

> *…conceptual understanding of the core knowledge of mathematics; procedural fluency in carrying out basic instructional routines; strategic competence in planning effective instruction and solving problems that arise while teaching; adaptive reasoning in justifying and explaining one's practices and in reflection on those practices; and a productive disposition toward mathematics, teaching, learning, and the improvement of practice. (National Research Council, 2010 p. 10)*

Educators Need Sufficient Time and Support to Learn

Bruce and Flynn (2013) suggest that powerful professional learning is classroom embedded, collaborative in nature, cyclical in design, and asset-based. Van Veen, Zwart and Meirink (2012) expand this research by characterizing effective professional learning as having a focus on subject and pedagogical content, including opportunities for active and inquiry-based learning, and being sustainable over time through an alignment with school and system policies within a supportive organizational culture. Research continues to underscore the importance of release time for teachers to learn together, and the effectiveness of job-embedded coaches to support educators in their classroom in assessment and evaluation in an on-going way (Garet, Porter, Desimone, Birman & Yoon, 2001; Bruce & Flynn, 2013;). This collective participation of educators improves the coherence in learning opportunities.

Educators Need Resources to Support Learning

Resources to support learning in mathematics need to be selected with consideration of several criteria. The selection of resources to support educators' and students' mathematical learning is underpinned by research that suggests that resources should build upon students' current knowledge; follow the natural developmental progression of mathematical understanding; teach procedural fluency with conceptual understanding; provide multiple opportunities for hands-on exploration, problem-solving and communication; and expose student to the major ways number is represented and talked about in developed societies (Bruce, Flynn, and Moss, 2012). Curriculum and resources that create opportunities for educators and students to engage in meaningful ways enhance both educator and student learning.

The Importance of Mathematics Coaches

Research has suggested that school-based mathematics coaches may be a vehicle to support the improvement of mathematics teaching and learning in elementary schools (National Research Council, 2001). The intent is to target educators' understanding and action, through partnerships with a knowledgeable colleague who possesses a deep understanding of mathematics pedagogical-content knowledge. These mathematical coaches would serve as an on-site resource for teachers to build collaborative professional development addressing mathematical content, pedagogy, and curriculum in an effort to enhance instruction and improve student achievement (Marzano, Walters, & McNulty, 2005). They would support deprivatizing mathematical practice within schools through developing and supporting the conditions that would allow educators to collaborate with each other in order to build mathematical capacity (Neufeld & Roper, 2003).

Bruce and Ross (2008), in a mathematics peer coaching study, found that when an educator receives positive and constructive feedback from a respected peer, there is even greater potential for enhanced goal setting, motivation to take risks, and implementation of challenging teaching strategies.

Sustained investment in mathematic coaches is crucial (Campbell & Malkus, 2011). Support for mathematics coaches includes a high degree of professional learning addressing mathematics content, pedagogy, and coaching prior to and during at least their first year in the coaching role. Over time, significant positive effects on student achievement can be attained as knowledgeable coaches gain experience and as educators learn and work together.

UNDERSTANDING THE MATHEMATICS LEARNING ENVIRONMENT

The learning environment, as suggested by many education researchers (LNS Monograph 27, 2012) is "the third teacher," which can either enhance or hinder the kind of learning that impacts students' potential to respond creatively and meaningfully to future challenges.

> *A classroom that is functioning successfully as a third teacher will be responsive to the children's interests, provide opportunities for children to make their thinking visible and then foster further learning and engagement. (Fraser, 2012, p.67)*

Educators have wondered what impact the mathematics learning environment has on student learning. We all come with different perspectives based on our diverse experiences with math learning and teaching. We turned to the literature to further our understanding of what a responsive mathematics learning environment looks and sounds like.

Aspects of the Learning Environment

There has been much written about the components that make up a responsive learning environment (Fraser, 1998; The Department for Education and Skills, 2006; LNS, 2011; 2012; Hannah, 2013; Taylor & Fraser, 2013). The Literacy and Numeracy Secretariat (2012) states that a responsive learning environment encompasses the physical and social-emotional environment including such components as student voice, collaborations, focus on solutions, real-world problem solving, and self-efficacy. The Department for Education and Skills in the United Kingdom explains that the learning environment is made up of three factors that overlap and impact each other: ethos, behaviours and routines, and the physical environment. Hannah (2013) identifies that the learning environment is affected by physical elements, emotional elements, and intangible elements such as the energy of the classroom, rules, and sounds.

Fraser (2012) defines the learning environment as "the social, psychological, and pedagogical contexts in which learning occurs and which affect student achievement and attitudes."

For the purposes of our study, the literature guided us to define a Responsive Learning Environment as one that includes three realms: the Physical Realm[1], the Choice and Voice Realm[2], and the Social and Emotional Realm.

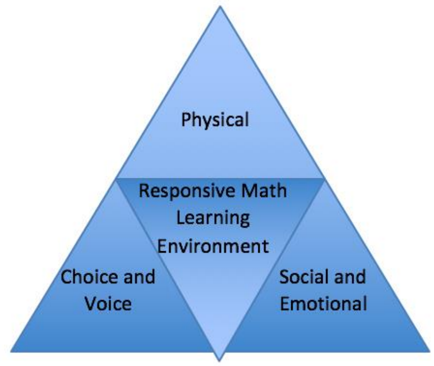


Figure 1- Responsive Mathematics Learning Environment Realms

Designing Mathematics Learning Environments

There is a plethora of research that examines the effect of the physical conditions of teaching spaces on students' engagement, attainment, attendance and wellbeing. Students' involvement in the process of creating their own environment can empower them, develop a sense of community and increase their motivation (Weinstein, 1979; Sundstrom 1987; McNamara & Waugh, 1993; Keep, 2002; Lackney & Jacobs, 2002; Earthman, 2004; Higgins et al., 2005;).

When considering the Physical Realm, educators look at the space of the classroom that promotes collaboration of group work as well as the space that permits quiet thinking and exploring of math. Active areas for inquiry, investigation and wonder are also considered to be part of the Physical Realm. Students' involvement in the process of creating their own environment and/or understanding the purpose of the resources available can empower students to develop a sense of community and increase motivation.

When educators consider the Social and Emotional Realm, students feel safer to take risks in math class so that they can make mistakes in order to try new ideas and strategies. Students also feel safer to revise their ideas and develop new mathematical understandings. When students feel supported by educators, they develop a more positive attitude toward math. Students feel they learn better in a "togetherness"[3] learning environment that gives them a sense of community where they share responsibility and endeavour (Kirsner & Bethell, 1992; Anthony & Walshaw, 2009). Furthermore, when students have more opportunities to independently explore and inquire, they feel more connected to the classroom community. In these classrooms, students not only hold more positive attitudes towards math, but they have higher performance in math (Yang, 2015).

---

[1] Components of the physical learning spaces impact students engagement, well-being and achievement.
Students' involvement in the process of creating their own environment and/or understanding the purpose of the resources can empower them, develop sense of community and increase motivation.

[2] Differing thinking, perspectives and ideas are valued as ways to deepen mathematical understanding.
Explore mathematics by choosing tasks, tools, methods and partners.

[3] A "togetherness" learning environment is where all members are supported by teachers and peers.
There is a sense of comfort in the classroom to take risks to make mistakes and try new things.

Educators recognize that students need to feel they have a voice and choice while learning. The Choice and Voice is important in creating a Responsive Math Learning Environment by encouraging differing thinking perspectives and ideas that are valued as ways to deepen mathematical understanding. Students have choice in exploring mathematics by choosing tasks, tools, methods and partners. As Boaler (2015), Suurtamm, Quigly, and Lazarus (2014) point out, students need to engage in tasks that challenge their current understandings and therefore have multiple entry points to meet the needs of the diverse learners in their class.

CONCLUSIONS

Our actions and decisions with respect to mathematics education must be grounded in the Full-Day Early Learning-Kindergarten program and the Ontario Mathematics Curricula for grades 1-8, 9-10, and 11 and 12. As well, a comprehensive vision of mathematics proficiency must guide and leverage mathematics discourse at all levels. The goals of mathematics proficiency are ambitious and necessary. Thus, thoughtful and methodical actions at all levels and by all education stakeholders should enable responsive, sustained and systemic attention to school mathematics.

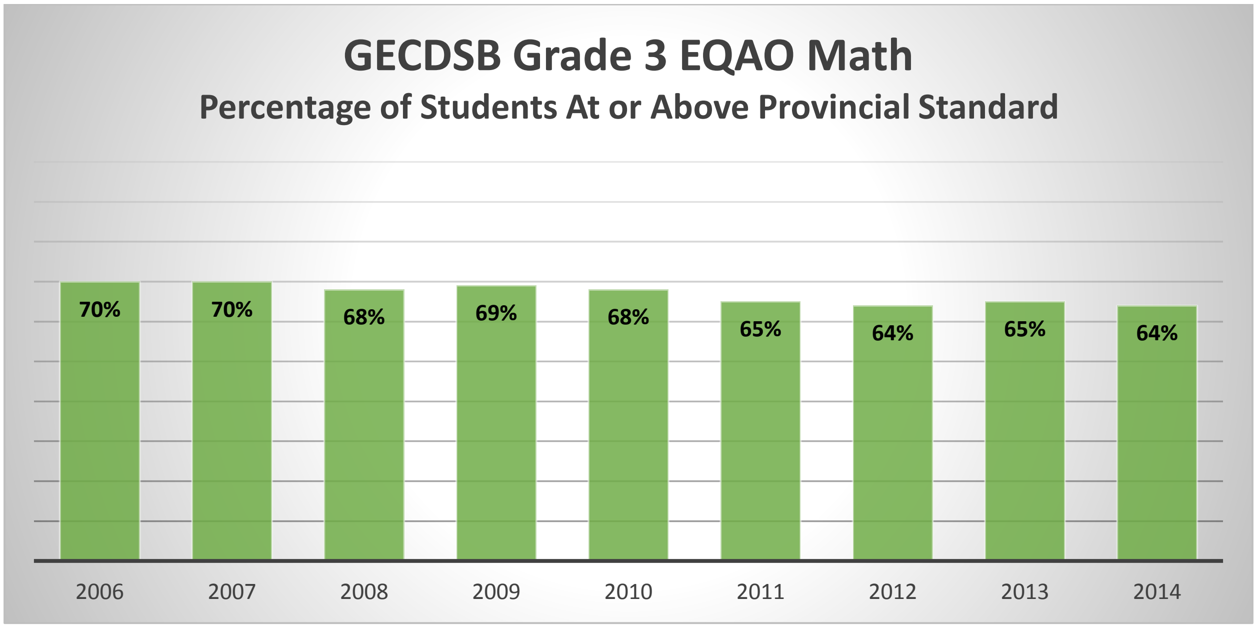
EQAO SYSTEM REPORT

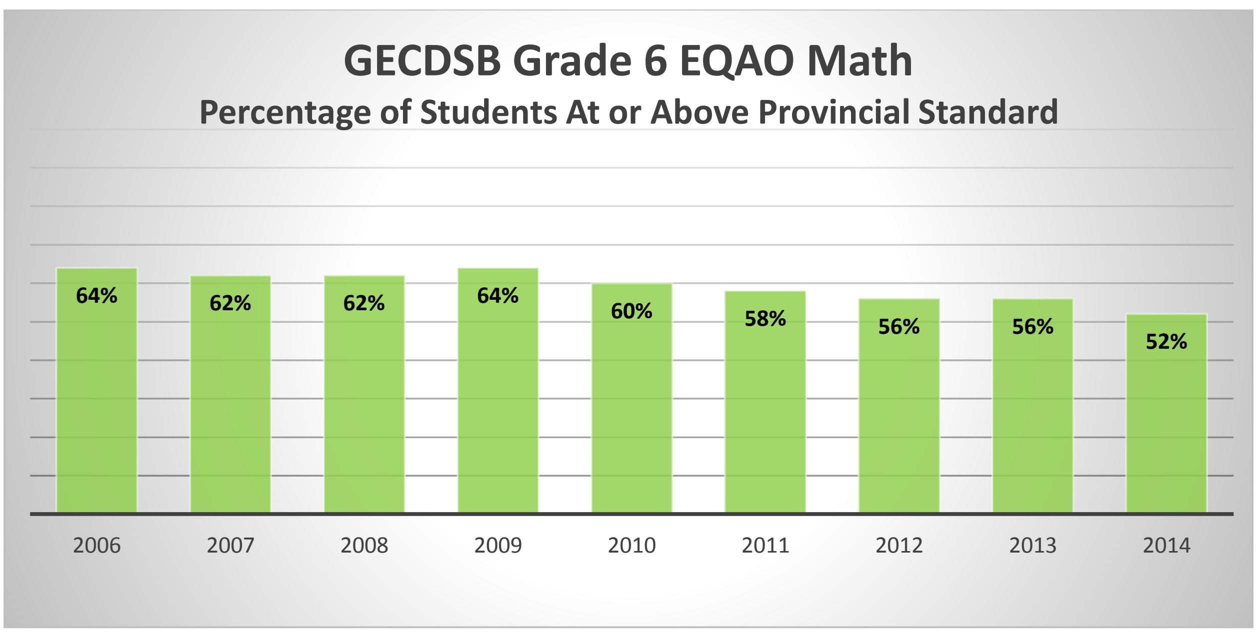
The Education Quality and Accountability Office (EQAO) measures the achievement of students across Ontario in mathematics in grades 3, 6 and 9. The assessments are based on the expectations found in the Ontario Curriculum at the end of each of the respective grades.

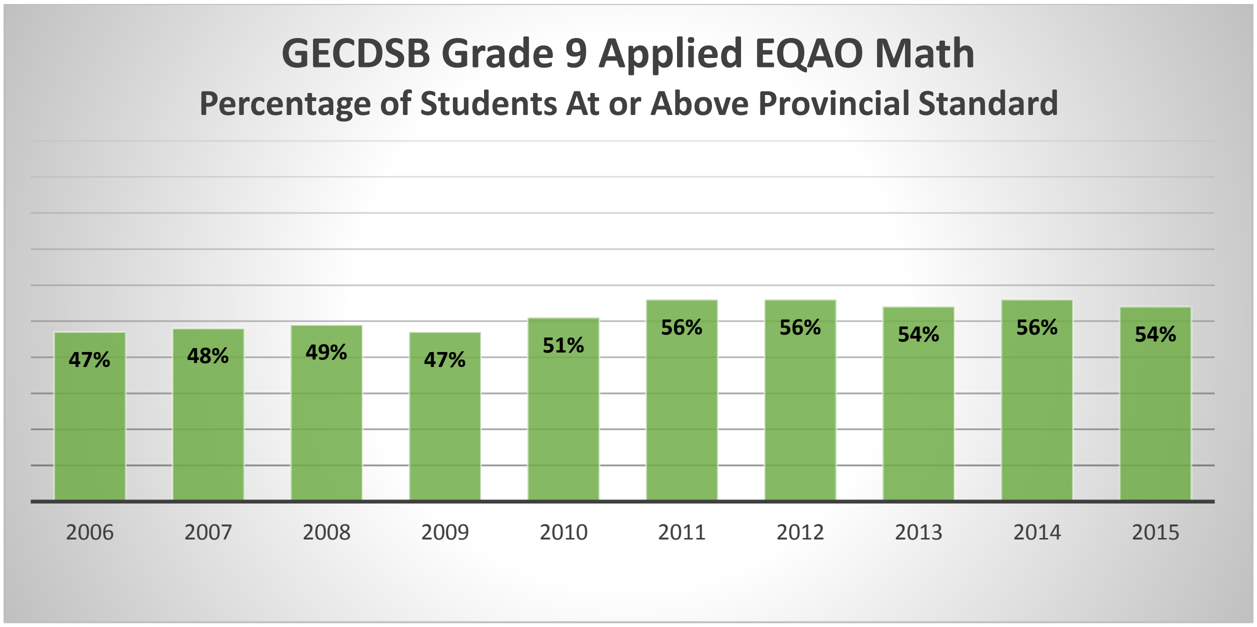
The EQAO Framework uses the following definitions to articulate an answer to the question, "What is Mathematics?"

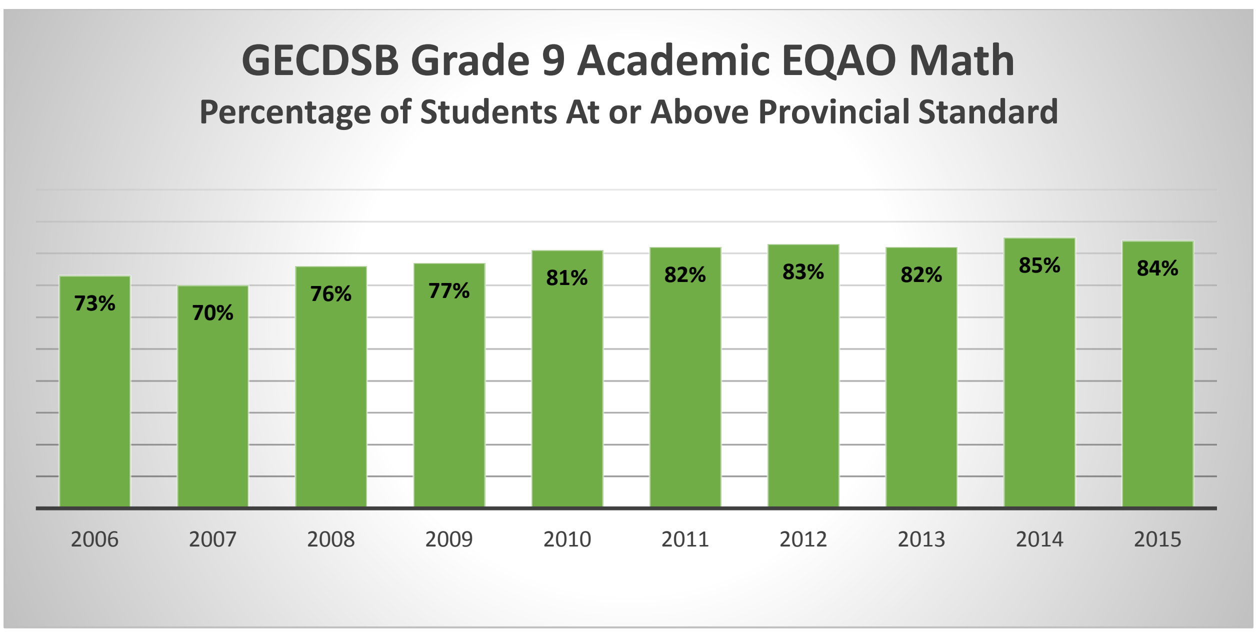
> Mathematical literacy is an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen (Organization for Economic Co-operation and Development, 2003).
>
> Achievement in mathematics goes beyond knowing mathematical facts and procedures; it also means being able to reason mathematically and to have the ability to interpret and solve mathematical problems (Artelt, Baumert, Julius-McElvany & Peschar, 2003).
>
> Mathematics involves many different processes. It is often defined as having the following five components:
>
> • *Conceptual understanding*— comprehension of mathematical concepts, operations and relations
>
> • *Procedural fluency*—skill in carrying out procedures flexibly, accurately, efficiently and appropriately
>
> • *Strategic competence*—ability to formulate, represent and solve mathematical problems
>
> • *Adaptive reasoning*—capacity for logical thought, reflection, explanation and justification
>
> • *Productive disposition*—habitual inclination to see mathematics as sensible, useful and worthwhile, coupled with a belief in diligence and one's own efficacy (National Research Council, 2001)
>
> These components are different aspects of a complex whole. They are interwoven and interdependent and cannot be easily separated.

The GECDSB mathematics results below indicate the percentage of students in Grade 3, Grade 6, Grade 9 Applied Mathematics and Grade 9 Academic mathematics who scored at or above the provincial standard. Ontario's provincial standard is equivalent to a "B" grade or 70%.






GECDSB Grade 9 Applied EQAO Math
Percentage of Students At or Above Provincial Standard
47%
48%
49%
47%
51%
56%
56%
54%
56%
54%
2006
2007
2008
2009
2010
2011
2012
2013
2014
2015


GECDSB Grade 9 Academic EQAO Math
Percentage of Students At or Above Provincial Standard
73%
70%
76%
77%
81%
82%
83%
82%
85%
84%
2006
2007
2008
2009
2010
2011
2012
2013
2014
2015

GATHERING VOICE: System Data Report

Overview

The data collection was done with the intention of uncovering the "voices" of stakeholders of the Greater Essex County District School Board. The data provides insight into the opinions and perspectives of students, educators and parents. The data plan was constructed, in part, by the Building Reciprocal Partnerships Ad-hoc Committee in consultation with research advisors and the Math Task Force. The data plan was designed to uncover baseline information regarding the perspectives and perceptions of various stakeholders.

Purpose

- to gain insight into the current perspectives of various stakeholders
- to ensure representation of the perspectives of stakeholders
- to inform the work of the Math Task Force
- to inform future work and learning within the GECDSB
- to support a democratic context of shared and collaborative partnership

Measures

Surveys and interviews were created in large part by the Building Reciprocal Partnerships Ad-hoc Committee and were refined by Math Task Force members. Both surveys and interviews were field tested with all sample groups (parent, student, educator, and administrator) in two schools. Results from the field testing informed further refinement of the measure and the administration protocols.

Surveys and Interviews

Sample Group	Dimension
Educators	• Math Mindset • Beliefs about Math Proficiency • Home-School Communication • Concepts of Partnership • Instructional Moves • Professional Development
Parents/Caregivers	• Math Mindset • Beliefs about Math Proficiency • Home-School Communication • Concepts of Partnership • At home math support
Students 4-12	• Math Mindset • Beliefs about Math Proficiency • Home-School Communication • Concepts of Partnership • In-class Support

Surveys were sets of statements that investigated the perspectives of stakeholders on various dimensions of mathematics attitudes, opinions and behaviours. The statements included a five-point Likert scale ranging from "strongly disagree" to "strongly agree" (See Appendix B).

School Samples

In all cases, participation in surveys and interviews was voluntary.

Twenty-five school across the GECDSB were part of the data collection. These schools were selected by the Math Task Force, based on several criteria. Fifteen of the twenty-five schools were selected based on the assignment of Special Assignment Teachers at these schools. An additional seven schools were added to the sample to include city schools, county schools and those with low and high mathematics achievement (EQAO). In the end, twenty-two elementary and three secondary schools participated in a range of measures.

Surveys

Student survey samples were taken from grades 4-12, one class per grade per school within the twenty-five selected schools. Student surveys were administered by Special Assignment Teachers and/or other GECDSB Program Department staff. A script was created for the administration staff and training was completed over two half-days. The administration of the surveys occurred in classrooms and participation was voluntary. An information notification was sent home to parents. Students had the option of not participating. In some cases, dependent on the student need (i.e. reading needs) or access to technology, survey administration was done with small groups of students. Outside of these special cases, the surveys were administered to entire classes. All surveys were administered electronically.

Parent surveys were available online through the GECDSB website. A notice was sent home to parents from all schools in the GECDSB inviting them to participate in the survey. In addition, information about participation was publicized through the GECDSB and school newsfeeds. The survey was available online from January 10, 2016 to January 30, 2016. Participation in the survey was voluntary. The survey was limited to one submission per IP address. Surveys could only be accessed electronically.

Educator surveys were made available by administrators to school staff to complete during staff meetings. The surveys were open to all staff to complete and participation was voluntary. Administration instructions were e-mailed to administrators along with the survey's electronic link. Surveys were administered electronically.

Administrator surveys were administered during system meetings to principal and vice-principal groups. Surveys were administered by program staff. All surveys were completed electronically.

Interviews

Participation in all interviews was voluntary. Parent and student participants were invited to participate. Interviews were conducted by Special Assignment Teachers or other GECDSB Program Department staff. A script was created for the administration staff and training was completed over two half-days. Interviews were approximately 40-50 minutes long. Participants could decline, skip questions, or end the interview at any time. Responses were validated either during or at the end of the interview to ensure accuracy of documentation. Consent was obtained before commencing the interview.

Student participants were selected by administrators and teachers. One "student of mystery" and one who excelled in mathematics were chosen from each division in grades 4-12. In elementary schools this would include 4 students in total. In secondary schools this would total 6 or more. A permission letter was sent home to parents with the details of the interviews. Upon commencement of the interview, assent was obtained by the students.

Parent participants were invited by administrators and teachers. A minimum of 3-4 parents were interviewed per school. In most cases the interviews were conducted one on one. In several cases, groups of parents were interviewed as a focus group of 5-9 people. Interviews were conducted, both face to face and over the phone, depending on the preference of the parent.

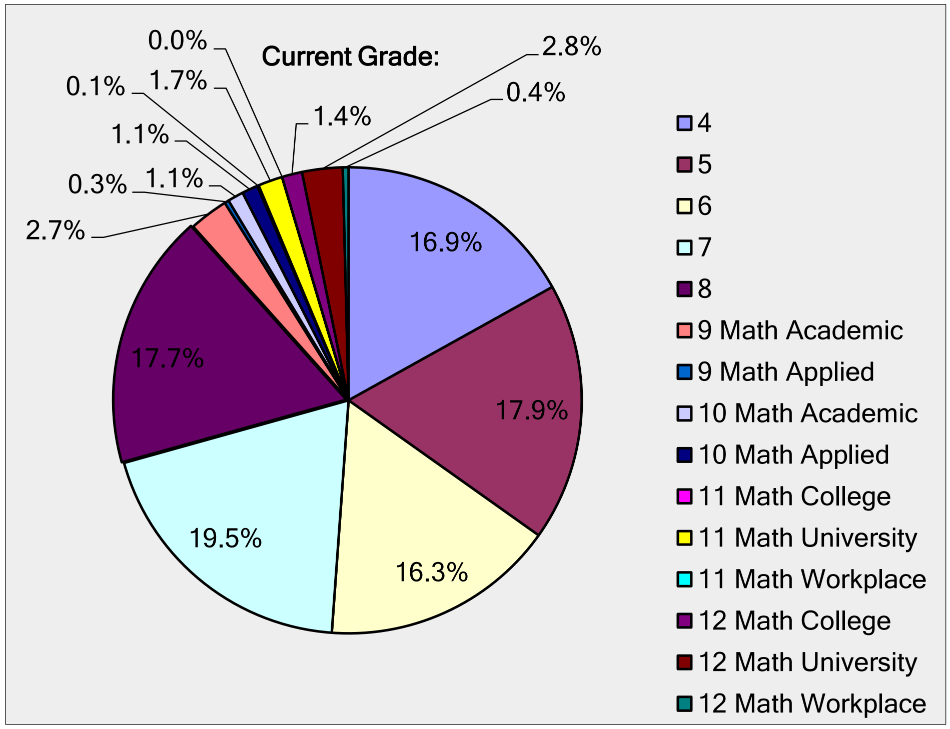
Educator participants were invited to attend interviews during the instructional day or at any point at which they could be scheduled. Occasional supply coverage was provided in order for educators to participate, although not all educators required or selected supply coverage. Many educators elected to participate during lunch, prep or after-school time. There were 5-6 interviews scheduled in a day and all interviews were conducted one on one. Interviews were conducted face to face.

Administrator participants were invited to participate via an e-mail invitation. All administrator interviews were conducted face to face. In most cases interviews were conducted one to one, but in some cases principals and vice-principals were interviewed together.

STUDENT VOICE: Survey Results

A total of 2162 students completed the survey. The proportion of students in elementary school was significantly higher than those in secondary (see Graph: Current Grade). Of the total respondents, 49.4% were female and 50.6% were male.

The perceptual data measured in the survey collected varying aspects of student attitudes, beliefs, opinions, and behaviours regarding mathematics.



Survey Section 1 - Mathematical Attitudes

The first set of survey statements measured students' attitudes toward mathematics and learning mathematics. In general, students demonstrated somewhat favorable attitudes toward mathematics. They also demonstrated similar responses when asked about their attitudes toward learning mathematics. The majority of students identified that math is useful and worthwhile.

Survey Section 2 - Defining Mathematical Proficiencies

Based on five survey statements that described the elements of mathematical proficiency, the student survey data suggests that respondents somewhat agreed with statements that identified mathematical proficiency.

Survey Section 3 – Home-School Connections

The majority of student responses identified that parents held positive attitudes toward math. The student responses also reported that their parents were sometimes able to support them with homework help. With respect to the use of online math resources, a limited number of students reported using online math resources at home.

Survey Section 4 – Mathematics Learning Perspectives

This set of statements examined students' perceptions of classroom mathematics learning. The majority of student responses reported that their teachers held high expectations of their learning in mathematics. The majority also reported that their classes discuss various ways to solve problems. Some students identified that they work collaboratively with a variety of students. The survey data also indicated that there is limited use of technology and manipulatives for math learning. In addition, some students identified that they are successful in math and a few identified that all students in their classes are successful in math.

STUDENT VOICE: Interview Results

There were a total of 61 students from grades 4-12 interviewed for this project. The interviews contained questions that were designed to explore various dimensions of mathematics learning experiences. All of the questions elicited a range of responses. The response were coded for themes and were analyzed to create a description of the perspective of grade 4-12 GECDSB students.

Section 1 – Doing Math

The first set of questions focused on attitudes, opinions and mindsets about mathematics. Students had a range of ideas to describe what it meant "to do math." The themes included concepts about the purpose and function of mathematics. Students described "doing mathematics" in terms of its application within the context of school (e.g. “solving problems in my class") or in terms of mathematics applied to real life or future profession (e.g. “you need it for your job.”)

The description of mathematics within the context of school included a wide range of responses including: dispositional ideas (e.g. "it is fun”), use of tools (e.g. “we use manipulatives”), procedures (e.g. “there are different ways to multiply”), and strategies and problem solving (e.g. “solving word problems”). The explanation of mathematics as applied to real life or professions included a more narrow range of ideas: everyday application (e.g. “you need to solve problems in everyday life”), or applied to a profession (e.g. “I want to be a builder so I will need math for that”). One student interviewed described "doing math" as a study or discipline of mathematics.

Section 2 – Being Good at Math

The next set of interview questions asked students about their ideas regarding success or aptitude in math. The student interview data demonstrated a description of “being good at math” in terms of performance skills and performance on tests and assignments. Many responses included references to performance on tests or evaluations (e.g. “getting answers on tests”) or performance in class contexts (e.g. “being right when we solve problems in class”). Others included references to performance skill: memorizing, explaining, communicating, and calculating. Student responses referred to being good at math as a set of social-emotional skills: persevering, learning from mistakes, solving problems, thinking, and enjoying.

Section 3 – Best Ways to Learn

The third set of questions asked students to identify the ways in which they best learn math. Student interview data, when asked how they best learned mathematics, provided a large variety of answers. In addition, the variation within the individual responses was also significant.

Section 4 – Self Concepts

Students’ assessments of their success at mathematics were closely tied to the way in which they defined "being good at mathematics." The data demonstrated that when students defined success in terms of performance skills and performance on tests and assignments, the descriptions of success were formulated in terms of performance or evaluation of performance (e.g. “I get good grades” or “I do well on tests"). Others defined their personal success in terms of social-emotional skills (e.g. “I never give up” or “I always do my best”).

Section 5 – Partnerships in Learning
Students were asked to share their ideas about partnerships in learning and identify their partners. In order to limit interpretation of the term *partnership*, it was defined in the interview as *people or groups of people working together toward a common goal*. In the data, most students identified their partners as their peers. Some identified teachers, siblings or parents. For the most part, students described their partnerships as collaboration with peers on classroom tasks.

Section 6 – Home-School Connection
This section asked students to reflect on the connections between home and school and the various communication methods used to support math. The data reflected a variety of communication tools used by teachers and parents. Overall, students identified homework, agendas, tests and assignments as the main methods of communication about mathematics. Students also identified phone calls, newsletters, blogs, websites, and apps as additional means of communication.

Section 7 – Student Voice
Students were asked if they felt they had a *voice* in their learning. In order to limit interpretation of the term, *voice* was defined as *having a say in your learning choices*. Student responses revealed an array of interpretations of voice. In general, the student interview data suggested that students struggled with the idea of "choices" but could identify the ways in which they may be given options (e.g. “I can choose to use manipulatives to help me” or “I can choose to draw something to solve my math problem”). The concept of "voice" seemed to be increasingly clear in the responses of older students. Students articulated that they may have a say in "how they learn" but choices about "what they learn" are dictated by an outside source, although they could not necessarily identify that source.

PARENT VOICE: Survey Results
A total of 870 parents completed the survey. Representation was demonstrated by nearly all GECDSB schools, with children in grade JK through 12.

The perceptual data measured in the survey collected varying aspects of parents’ attitudes, beliefs, opinions, and behaviours regarding mathematics.

Survey Section 1 – Attitudes toward Mathematics
The first set of questions measured parents’ attitudes toward mathematics. In general the parent respondents displayed positive attitudes toward mathematics.

Survey Section 2 - Defining Mathematical Proficiencies
Based on five survey statements that described the elements of mathematical proficiency, the majority of parent responses demonstrated agreement to all of the statements defining mathematical proficiency.

Survey Section 3 – Home-School Connections
The next set of statements examined the connection between home and school. The majority of parent respondents indicated that they encourage positive attitudes toward education and set high expectations for their children. They indicated that they feel as though they can somewhat support mathematics learning at home and identified that they make limited use of technology for mathematics learning at home. The majority of parents indicated that they felt partnerships with the schools were important. The survey data also suggested that some parent respondents demonstrated positive attitudes regarding the

communication between home and school but that opportunities to further develop partnerships would be beneficial.

Open-Response 1: "Please list the various online math supports that you and/or your child uses."
There were 380 responses in this section. Parents identified an array of online math supports which encompassed two main categories: videos and demonstrations; and problems and practice. Parents were able to identify specific apps, sites and software. Interestingly, very few parents made mention of any specific online technological tools (e.g. graphing programs or online manipulatives).

Open-Response 2: "List the various ways in which you communicate with your child(ren)'s school/teacher about math."
There were 467 responses in this section. Parents identified a range of communication tools. The most frequently used tool is the take-home agenda. Parents also identified communication tools which fit in to two main categories: one-way communication tools (e.g. newsletters, blogs, online posts) and two-way communication tools (e.g. agendas, phone calls, visits, etc.).

PARENT VOICE: Interview Results

There were a total of 63 parents interviewed for this project. The interviews contained questions that were designed to explore various dimensions of parental mathematics experience. All of the questions elicited a range of responses. The responses were coded for themes and were analyzed to create a description of the perspective of parents within the GECDSB.

Section 1 – Doing Math
The first set of questions focused on attitudes, opinions and mindsets about mathematics. There were three main categories of purpose and function of mathematics: everyday application, professional application and expansive mathematical thinking and processes.

The majority of parents described mathematics in terms of a functional life skill (e.g. "you need can't get through life without it" or "you need it for cooking or if you are renovating your home"). Others described mathematics in terms of its application to various professions (e.g. "it is a requirement for almost all employment"). The last category of responses described various types of mathematical thinking or processes (e.g. "it is reasoning and thinking analytically" or "it is logical thinking and problem solving").

Section2 – Being Good at Math
The next set of questions asked parents to describe success or aptitude in math. The interview data demonstrated that the descriptions of what it meant to "be good at math" closely resembled the themes identified as the purpose and function of mathematics. In those interviews in which the responses reflected mathematics as a functional skill, parents heavily identified that basic facts were an important indicator of success. Other descriptions included speed and accuracy as an indicator of being good at math, and still others identified social-emotional skills like resilience, perseverance and determination as examples of what it meant to be good at math.

Section 3 – Self Concepts
The parent interview data indicated varied personal assessments of their ability in mathematics. The data included specific examples, explanations or instances that demonstrated their self-assessments. Some interviews described a significant "no" response with strong negative emotions to mathematics (e.g. "No.

I'm not good at it. Hate it and worry about helping with math"). Other responses identified more positive self-perception in terms of relevant applications of mathematics, but nevertheless indicated that they did not feel they were proficient or comfortable (e.g. "I can do the basics, budgeting and what I need for work but I am no good at fractions"). Yet another group of responses identified an ease and confidence with experiences in mathematics and mathematics in general (e.g. "I was always good at it. I just think that way").

Section 4 - Partnerships
These questions asked parents to describe their current partnerships and elements of ideal partnerships. The parent interview data reflected a range of responses about partnerships. Many partners in learning were identified including educators, administrators, other families, children, and community members. The data demonstrated that close partnerships were identified among those who had close relationships (e.g. "I volunteer at the school. I see the teachers all the time and know them well. They are my partners. The principal is too, I work with him through school council").

Section 5 - Home-School Connections
The interview data demonstrated a range of responses regarding the connection and support of mathematics learning between home and school. Many communication tools were identified including agendas, face to face meetings, technological aids (e.g. apps, blogs, websites), and assignments sent home. The responses indicated, that at times they felt that mathematics learning could be better supported between home and school, and others felt it was very well supported.

EDUCATOR VOICE: Survey Results

A total of 912 educators completed the survey. The respondents included educators in varying roles within the GECDSB. The majority of respondents were in the elementary panel with the largest group being primary and junior educators.

Of the total respondents, 709 indicated that they were currently teaching math. Of the total respondents, 7.7% indicated that they had additional qualifications in mathematics and 6.7% indicated that they had a mathematics teachable qualification.
Respondents varied in their years of teaching experience, with the majority of respondents indicating that they had over 10 years of experience.

The perceptual data measured in the survey examined varying aspects of educators' attitudes, beliefs, opinions, and behaviours regarding mathematics. The following analysis is based on the total number of educator responses. Results would vary if the sample included only those who were teaching mathematics. Where appropriate, any differences in these two samples is noted.

Survey Section 1 – Attitudes toward Mathematics
The survey data indicated that the majority of educators demonstrated favourable attitudes toward mathematics and mathematics teaching. The data also identified that most educators felt they were good at math and felt comfortable teaching math.

Survey Section 2 - Defining Mathematical Proficiency
Educator survey data demonstrated agreement with the statements defining mathematical proficiency. The results for those educators who indicated they were teaching math were slightly higher in all of the statements. Overwhelmingly, educators indicate that they believe mathematics is useful and worthwhile.

Survey Section 3 – Home-School Connections
Educators indicated that they consistently communicate with parents. The data also indicated that specific communication of mathematics could be improved. The survey data also identified that educators valued partnerships with parents and sometimes engaged parents as partners in learning.

Survey Section 4 – Mathematics Teaching and Learning
This set of statements examined educators' perceptions of classroom mathematics learning. The survey data identified that educators felt that their students were somewhat prepared to learn mathematics. Educator responses indicate use of a variety of math resources and consistent use of manipulatives, but limited use of technology for mathematics learning. In terms of assessment, the majority of responses indicated *consistent* assessment *for* learning and *constant* assessment *of* learning. The high rates of agreement to these statements raise questions about these statements themselves, how they are being interpreted, and the general practices of assessment in mathematics. Further investigation is necessary.

Survey Section 5 – Professional Development
These statements surveyed educators' professional development preferences and experiences. The survey data reflects somewhat positive perceptions of professional development. Responses indicate that some educators are not presently involved in mathematics professional development. In addition, some of the responses identify a preference for professional development in math content and pedagogy. These responses are slightly higher when examining educators who are currently teaching math.

Open-Response 1: "The Professional Learning Structures I have found most effective are..."
There were 413 responses to this open-response section. Educators identified a range of structures that supported professional development in mathematics. The structures that were most frequently identified were PLCs, Book talks, Collaborative Inquiry Learning in Math (CILM), and Middle Years Collaborative Inquiry (MYCI). Educators indicated that structures were most beneficial when they consisted of small groups of professionals with relevant and similar foci. Educators also identified the utility of various mathematics experts in their professional learning.

Open-Response 2: "List the various methods that you use to engage parents in their child(ren)'s math learning:"
There were 600 responses to this open-response section. The most frequently mentioned methods of engaging parents were agenda, newsletters, blogs, and other online posting methods. Educators were overwhelmingly able to articulate various methods of communication. Further analysis of this open-response data will provide insight into the methods and processes of parent engagement.

EDUCATOR VOICE: Interview Results
There were a total of 127 educators from the elementary and secondary panel interviewed for this project. The interviews contained questions that were designed to explore various dimensions of mathematics learning experiences. All of the questions elicited a range of responses. The responses were coded for themes and were analyzed to create a description of the perspective of educators within the GECDSB.

Section 1 – Doing Math
The first set of questions focused on attitudes, opinions and mindsets about mathematics. The interview data conveyed a range of ideas about what it meant "to do math." The themes included the development and application of skill sets related to mathematical procedures, concepts, problem solving, thinking and disposition. Interestingly, some responses contained narrow descriptions (e.g. “math is about the rules of numbers”) and others were broad and expansive (e.g. “math is problem solving, it is a language, a way of thinking and communicating. Doing math is part algorithms and computations, part problem solving, part strategizing and a whole lot of ways of thinking”).

Section 2 – Being Good at Math
The next set of interview questions asked about ideas regarding success or aptitude in math. The interview data demonstrated description of success in mathematics in terms of skill sets and applications of skills sets. Success in mathematics was described as mastery or application of skills: communication, procedural, conceptual, problem solving, thinking, and dispositional. In general, the application of these skills was seen as “real world” or “real life.”

Educators overwhelmingly identified that their perceived students would identify success in mathematics in terms of correctness/accuracy or assessments/evaluations. Some respondents included descriptions about social-emotional skills like “perseverance” in their assessment of student responses.

Section 3 – Self Concepts
These questions asked educators about their personal assessments of their aptitude in mathematics. Educators’ assessments of their ability or success at mathematics varied. The questions were interpreted three ways: success/ability in mathematics teaching; or success/ability in previous mathematics education; or success/ability in mathematics in general.

Those who interpreted the question as success/ability in mathematics teaching discussed comfort or ease with mathematics teaching in the context of their grade or the grades they have taught. Many respondents indicated a lack of comfort with the grades significantly above or below their assigned grade level (“I am comfortable in grade 4. I have taught it for years, but I would have to really go back and figure things out if I was going to go teach grade 8.”) The group of educators who interpreted the question as success/ability in previous mathematics education generally highlighted their successes and achievements in mathematics throughout their education (“I was always pretty good at math. I did well in high school all the way through and then in university”). The third group interpreted the question as success/ability in mathematics in general or as applied to daily living and provided an array of responses highlighting how they are successful in some application but not in others.

Section 4 – Partnerships in Learning
These questions asked educators to describe their current partnerships and aspects that contributed to their vision of an ideal partnership. In order to limit interpretation of the term *partnership*, it was defined in the interview as *people or groups of people working together toward a common goal*. In general, educators were able to identify one or more partners. There were a few respondents who did not feel they had any viable partnerships.

Educators’ partners included co-workers, administration, parents, students, central office staff and support workers. These lists could be categorized as “inside” and “outside” of the school. In general, the descriptions of ideal partnerships were most closely tied to those “inside” the school.

Section 5 – Home-School Connection
There was an extensive range of responses in the ways in which educators supported math learning between home and school. The categories included one-way or two-way communication via information sharing (“agendas, newsletters, blogs, twitter, letters, apps, web-sites”), or one-way or two-way communication via collaborative learning (“take home math problems, family math problems, game nights, online parent/child math learning, family math nights, drop-in sessions”).

Section 6 – Professional Development
This set of interview questions asked educators about their professional learning experiences, needs and preferences. Educators identified specific conditions which supported rich professional learning. The themes include relevance, collaboration, co-learning, guidance, resources and time. Educators indicated they are interested in participating in professional learning that supports both math content and math pedagogy. The responses stressed that the learning is most beneficial when it is relevant to their specific student learning needs.

ADMINISTRATOR VOICE: Survey Results
A total of 89 administrators completed the survey. The respondents included Principals and Vice Principals in the GECDSB elementary and secondary panels. The majority of respondents indicated that they had over 10 years of teaching experience. The respondents also indicated that the majority had over 6 years administrative experience.

Of the total respondents, 10% indicated that they had additional qualifications in mathematics and 11% indicated that they had a mathematics teachable qualification.

The perceptual data measured in the survey examined varying aspects of administrator attitudes, beliefs, opinions, and behaviours regarding mathematics. The following analysis is based on the total number of administrator responses.

Survey Section 1 – Attitudes toward Mathematics
Administrator survey data generally demonstrated positive attitudes toward mathematics and teaching mathematics. In addition, the data indicated that the majority of respondents feel as though they are good at math.

Survey Section 2 - Defining Mathematical Proficiency
The data identified that the majority of administrator responses agreed with the statements defining mathematical proficiency.

Survey Section 3 – Home-School Connections
The survey data indicated that parent engagement is important and valued. The data also reflected that improvements could be made with respect to communication with parents about math.

Survey Section 4 – Mathematics Teaching and Learning
Administrator survey data reflected that they sometimes observe students engaged in mathematics problem solving. All other observations about mathematics teaching and learning were varied.

Survey Section 5 – Professional Development
The majority of administrator responses indicated that they are currently supporting mathematics professional development in their schools. Some administrators felt as though they could effectively lead math teaching and learning in their schools and most felt that they required additional resources to help support mathematics teaching and learning. The majority of responses indicated they would like additional professional development in mathematics.

Open-Response 1: "The Professional Learning Structures I have found most effective are:"
There were 69 respondents to this section. Administrators were able to identify several structures that were effective in supporting their professional learning including Student Work Study, Collaborative Inquiry Learning in Mathematics, Middle Years Collaborative Inquiry, and embedded Professional Learning Communities.

ADMINISTRATOR VOICE: Interview Results
There were a total of 21 administrators from the elementary and secondary panel interviewed for this project. The interviews contained questions that were designed to explore various dimensions of mathematics learning experiences. All of the questions elicited a range of responses. The response were coded for themes and were analyzed to create a description of the perspective of administrators within the GECDSB.

Section 1 – Doing Math
The first set of interview questions focused on attitudes, opinions and mindsets about mathematics. The majority of responses described mathematics in terms of an application to and for "life" (e.g. "Math is everywhere" or "You need math to be successful in life. You need to be able to solve problems and use it in life and work").

Section 2 – Being Good at Math
The next set of interview questions asked administrators to identify what defined success or aptitude in math. The interview data demonstrated a description of success in mathematics in terms of skill sets and utility of skill sets. The interview data overwhelmingly identified that the application of mathematical skills was a measure of what it meant to be good at math.

Section 3 – Self Concepts
The data reflected that administrators' assessments of their own ability or success in mathematics varied. The responses generally demonstrated that success in mathematics was determined by an application of mathematical skills and so the responses indicated confidence in applying mathematical skills.

Section 4 – Partnerships in Learning
In order to limit interpretation of the term *partnership*, it was defined in the interview as *people or groups of people working together toward a common goal*. The interview data reflected a rage of partners including parents, staff, international partners, administrative partners and colleagues, and students. The data reflected that partnerships were valuable and there were many conditions which supported positive partnerships including trust, collaboration, sharing of work load, mutual respect, and shared vision.

Section 5 – Home-School Connection
Administrators were asked to share their ideas about partnerships in learning and to identify their partners as well as their vision of ideal partnerships. The interview data indicated that a connection between home and school was valued. The responses identified a range of means that supported math learning between home and school including math nights, classroom communication, and school communication. The data indicated that the connection between home and school could be better supported by building common understandings, visions and consistent communication (e.g. “We just started adding things to the newsletter. We tried problem of the week on the website”).

Section 6 – Professional Development
This set of interview questions asked administrators to identify their preferred professional development structures. As well they were asked for their perspectives regarding the supporting of school-based mathematics professional learning. Administrators were asked if they felt they could fully support mathematics learning in their schools. The interview data indicated two categories of responses (e.g. “yes” and “yes, but I require support”). The data reflected a range of structures that were helpful in supporting mathematics professional learning. The structures included the following themes: relevancy, consistency, school-based, small group, collaborative and inquiry-based.

Conclusions
Greater Essex County District School Board understands the need to engage all stakeholders in creating conditions for improvements in student achievement. The work of the Math Task Force was enhanced through the insights and representative voices of these stakeholders and this input influenced the formation of many of the considerations posed by the Task Force. A commitment to an ongoing process of data gathering, monitoring, and stakeholder engagement will be integral to future improvements and emphasize a joint commitment to improving outcomes for all our students.

MATHEMATICS LEARNING ENVIRONMENT REPORT:
Methods, Data Analysis and Limitations

Methods

The Responsive Mathematics Learning Environment committee was interested in visiting classrooms in our system to better understand what the literature has identified as a Responsive Math Learning Environment. The committee was composed of three elementary school principals, a board trustee, a parent, an elementary teacher, and two program department staff members. The committee contacted teachers at 16 elementary schools and one secondary school in our system. The purpose of the classroom visits was to gain a deeper understanding of the mathematics learning environment (Figure 1) through observations, which will lead to board wide learning. The underlying assumption for the visit was that the team, teachers and students would work together to create some new knowledge.

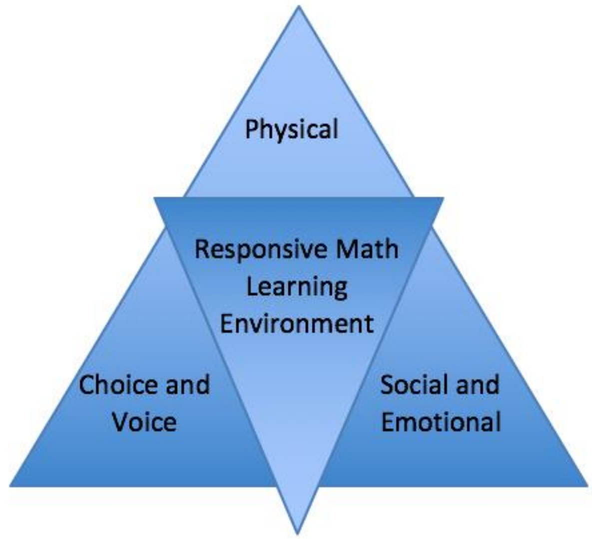


Figure 1- Responsive Mathematics Learning Environment Realms

The teachers were emailed directly by the superintendent of program inviting them to consider volunteering to host a visit from our Learning Environment committee. Once the teachers sent their expression of interest, they were emailed a link to a brief student survey (See Appendix C). The Responsive Mathematics Learning Environment survey was composed of 12 questions, 4 questions in every realm. The individual class results will later be shared with all teachers who expressed an interest in hosting a visit.

The Classroom Observation Visit began with a pre-meeting with the teacher. The purpose of the meeting was to share the details of the visit as well as to build trust. The visit was comprised of 30 minutes of observation followed by 45 minutes debrief. During the observation period, the visitors entered the room with the three realms fresh in their minds. The visitors observed the physical environment and documented students' conversations during work to illuminate the Choice and Voice and Social and Emotional Realms. The visitors were asked to refrain from talking to the teacher during the observations. All the questions directed to students were based on immediate classroom observations.

After the classroom visit, the observation team reflected upon the documentation and prepared for the debrief. The team reviewed their data to ensure an asset-based debrief. Each member highlighted asset based observations that connected to a responsive mathematics learning environment and transposed the highlights onto post-it notes. The notes were organized in themes that emerged from the observations. Wonderings were derived from the themes, and formulated into questions for the teachers and students to consider answering during the debrief.

The debrief sessions were intended to be an open conversation between the team, teachers and students. Half day release was provided for each teacher for the observation and debrief sessions. At each site, each teacher chose four students to be present for the debrief session. The questions were asked and as teachers and students responded, notes from the debrief were documented onto chart paper that was visible to all students and teachers during the session. All original documentation of the observations and the debrief were given to the school for the possibility of informing future work and sharing with the students and educators.

Findings and Discussion

The response to the email sent out was overwhelming considering the time limitations. We were contacted by 18 teachers. All teachers received individualized links to the student survey. A total of 272 students completed the survey. The following table indicates the percentages obtained from the combined total for "often" and "almost always" responses in each realm.

Realm	**Social & Emotional**	**Choice & Voice**	**Physical**
Combined total %	71	63	64

Thematic analysis is a method for identifying, analyzing, and reporting patterns within data (Braun & Clark, 2006). This type of analysis is flexible and accessible to researchers with results being available to all. Emergent themes from the debrief session were based on the highest frequency of occurrence of ideas and opinions. These themes were categorized under the three realms of the Responsive Mathematics Learning Environment.

Physical Environment

Themes	Noticings from Classroom Visit	Students' Conversations during Observation and Debrief
Space	• Meeting areas • Quiet areas • Independent areas • Small group areas • Large group areas • Working together • Working alone	"We learn at the carpet and our desks." "Sometimes we work in groups of 3 and 4. All of the differences go together to make us understand better." "Too many partners can be distracting."
Visual Aids	• Charts and notes around the room • Learning goals and success criteria • Math strategies, formulae • 100 charts, number lines • Connection between what is on walls and what students are learning	"Math ideas are everywhere to help me when I'm stuck." "Learning goals make us much more organized. Seeing what we need to know before and after helps." "Those things on the walls build a community and help you if you are stuck."
Student Generated Ideas	• Student generated ideas and strategies • Student work posted • Student writing on the charts	Student work displayed makes the students feel: ○ "proud to have our work up on the wall" ○ "proud to have your work up but also nice to help others to put their work up" ○ "challenges us to dig deeper" ○ "motivates me" ○ "I feel more comfortable because it's what we did and it's our ideas." ○ "If you put something on the wall, maybe it will help someone else and they will try that idea." ○ "I put things up on the wall that I can understand, things that I can do in my head and have learned."
Accessible Manipulatives & Math Games	• Manipulatives help us • Math games reinforce • Students use math tools when needed • Manipulatives on the shelves; labelled • Manipulatives available throughout the room	"Manipulatives helps us learn math and we are allowed to use them."
Technology	• Apps (to practice and share learning) • Games (the "during/action" phase and after assigned work is done) • Sharing with peers (on walls, during consolidation) • Sharing with parents (blogs, websites)	"We share our work [on KidBlog] and get feedback." "Technology helps with sharing our learning." "Because of this [website] my parents ask more about my other classes too." "The best part of our classroom is they teach us how to use apps."

Choice and Voice

Themes	Noticings from Classroom Visit	Students' Conversations during Observation and Debrief
Choice in a Variety of Options	• Choice of partnerships • Choice of resources • Choice of strategy • Choice of task • Choice of where to work	"We have a choice to work with someone who struggles to help them." "If you worked with a person before and you didn't co-operate, you politely say no I didn't work well with you.'" "We can use anything you want to get the answer." "We learn different strategies by working with other students." "We often have two options. Start with the one you are comfortable with." "Freedom to move where you work best."
Recognition of Self	• Students recognize what style of learning best meets their needs • Students know what choices are best for their learning and teachers trust students	"Sometimes saying it out loud, the way it sounds, helps me remember." "I find it easier to learn when I see 2 ways or more to figure something out. Multiple ways let me choose the best way for me to do it" "Do what you need to do." "I asked to sit near the front. I feel more involved."
Collaboration, Discussion, Teaching, Questioning	• Collaborating with others to share and discuss ideas • Teaching others • Questioning	"We talk to each other and understand what the person is thinking." "It's kind of like having your brains connected." "Sometimes we don't listen and then the partner helps us if we missed out on things." "I know I have it when I can teach it to someone else." "I learn from some other people." "I'm not embarrassed to ask questions." "I ask my teachers and my friends."
Time	• Time to think • Time to talk • Time to learn	"I have time to think through a question." "We have time to discuss our answers." "Chance to redo our tests, chance to study to learn…will have a chance, always a second chance." "It's okay if you don't get it right."
Engagement	• Focused during math time • Accountable talk • Focused during transition times • Using tools to solve task • Draw names to come to debrief meetings	"We have time to focus because we are **IN** it or how else would you know? I think we bring information because we are **IN** the learning."

Social and Emotional

Themes	Noticings from Classroom Visit	Students' Conversations during Observation and Debrief
Community of Learners	• Pride • Team atmosphere • Class of learners • Give high fives • Safe environment to talk, ask for help and take risks	"Miss makes it comfortable so we're more in tune to what we're doing." "We learn better when we are in groups." "There's a lot of people around us to help." "I learn from others who teach me." "After every test, (we) change seats. Every time we meet someone new…positive effect on both."
Growth Mindset	• Strong focus on learning through mistakes • Opportunity to continue learning by using multiple means of feedback • Develop confidence and belief in themselves that they can learn math	"When I'm struggling, I ask for help from the teacher and peers." "Chance to redo our tests, chance to study to learn…will have a chance, always a second chance." "Class promise says we learn from our mistakes." "If one of my classmates is having a fixed mindset day, I try to make them feel better, I say take that piece of paper and throw it in the recycle bin." "My confidence helps me learn math." "When I'm nervous I have a fixed mindset. When I am happy, then I feel I can do it."
Grit	• Persevere • Keep Trying	"The more you do it the more you understand it" "When you have done it a few times, your brain gets used to it. The first time you will be confused. Then the more you do it the more you understand it" "It gets easy."
Clear Expectations	• Clear learning goals with strategies for success • High expectations	"She [the teacher] want us to succeed and learn."
Seeing other strategies	• Use different strategies while working together • Partner who helps uses a different strategy	"We learn different strategies by working with other students."

Throughout the analysis of the qualitative data, we noticed an overlap between the different realms. The realms clearly do not stand alone, but they are interrelated. Educators understand that each realm does not exist in isolation from the others. The correlation value between two of the questions in the survey ("In this class, I know that I can find help when I need it" and "In this class, I know how to use materials to explore math") was 0.98. This significant correlation shifts our attention to the interrelationship between the Physical Realm and the Social and Emotional Realm (Figure 2). A Responsive Learning Environment is created as each realm overlaps, connects and impacts the others to make the classroom more stimulating, challenging and safer (Department for education and skills, 2013).

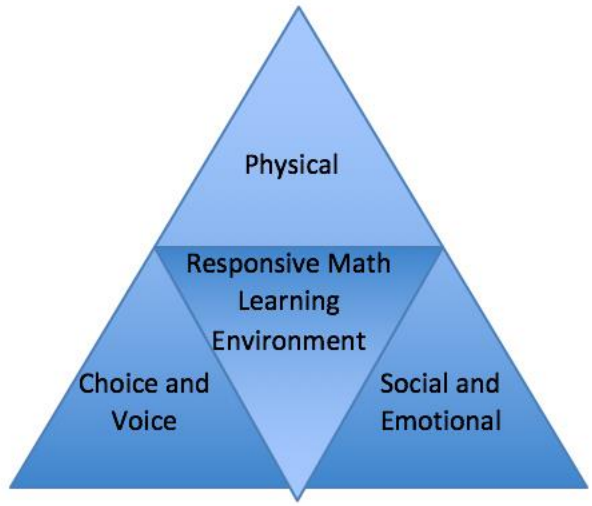


Figure 2- New Responsive Mathematics Learning Environment

Much of the recent math literature speaks to the importance of engaging students in an inquiry approach to learning mathematical concepts (Fosnot & Dolk, 2001; LNS, 2012, 2011; Suurtamm et al, 2015; Boaler, J., 2015).

We are left wondering if we saw an inquiry approach to learning math, and how an inquiry approach is similar to or different than a problem-based approach or open-ended questions in math. In addition, if the Social and Emotional and Choice and Voice Realms of the Responsive Mathematics Learning Environment are very important, then how can we investigate the effects of non-semestered math classrooms on students' well-being and achievement. Researchers such as Muse (1998) and Williams (1999) agree that the traditional schedule, where a math period is no longer than 50 minutes, is far more effective in covering the appropriate amount of material, keeping students focused during the entire class period, and improving students' performance in math.

Limitations

There were many limitations that were identified in this study.

1. Time

Time appears to be the main factor that limited our study.

A. **Email Request**: We wonder if time impacted the number of teachers who volunteered to host a visit. The email request was sent out at the time where teachers were in the middle of writing report cards. Also, the email went out a day earlier than the other Math Task Force data collection email and this may have caused some misunderstanding as to what survey, interview or classroom visit our educators were considering.

B. **Number of Classrooms Visited:** Due to time, we decided to visit schools where two or more teachers invited us in so that we could visit more classrooms in one day without having to include travel between schools. We were not able to visit all teachers who volunteered to host a visit; we had to respectfully decline five teachers.

C. **Early Years Classrooms:** We recognize that there is much to learn from an Early Year's responsive mathematics learning environment but we did not have the opportunity to visit one classroom yet. We did have two Early Years educators invite us to their classrooms, but we were not able to schedule a visit within the timeline. We are curious how the debrief will unfold with our youngest learners.

D. **Semester Schools:** When we sent the email to our elementary schools, our secondary schools were preparing for exams and students were completing their Final Summative Evaluation so we decided to wait to send out the email until semester two. When semester two began we decided it was best to give our teachers and students time to acclimatize for their new classes before sending the email. We have not emailed them yet.

E. **Observation Time:** We started off observing for 75 minutes but felt that we saw plenty in less time so we reduced our observation time to 25 minutes. We recognize that we can't see everything that happens in a responsive math learning environment in this time period. It is important to note that much of our observation time took place during the 'action' phase of the math lesson where students were working independently or in small groups on a math task.

2. Documentation

We pay attention to what we are curious about, and then we write down our wonderings. These documented observations become the springboard to our debrief conversation with the students and teachers. In an attempt to neutralize this limitation, we began asking this question at the end of the debrief: "What did we miss?"

CONSIDERATIONS

Considerations	Areas of Impact					Additional Details	Action Items	Timeline
	Home or Community	Classroom	School	System	Ministry			
1. A district-wide model for professional learning should be developed, communicated, supported, and celebrated. This model should be multi-year and focused on content learning, pedagogical learning, and pedagogical-content learning. The model should focus on deepening proficiency as defined in the GECDSB Vision for Mathematics and be differentiated according to the needs of schools, departments and educators. The model should serve the goal of improving math learning for every student in every classroom.	X	X	X	X	X	This model will encompass Elementary and Secondary panels. The model should align with the resources and funding models provided by the Ministry of Education and should include input from provincial math leads and Ministry partners. This model will include consideration for embedded Professional Learning Communities and the continuation of School-Based Learning models, as well as the continued development of networked learning opportunities for elementary and secondary teachers to learn across schools and panels. This model will consider the supports needed for secondary department heads to lead math learning in their departments. This model will also consider the continuation of existing collaborative inquiry structures, including those in partnerships with the University of Windsor and the Ministry of Education (e.g. CILM and SSSSI). This committee will also consider the creation of a voluntary "School Math Lead" position in each Elementary school. Each school will submit a person for this role. This person will receive additional specific supports and learning and will be a point of direct communication between Central Office and the school in the support of math learning for educators and students.	1. Establish representative committee 2. Develop initial plan for 2016. 3. Develop long-term plan for 2016-2022	Initial plan by June 2016, implementation beginning September 2016. Long-term plan complete by February 2017.

Considerations	Areas of Impact					Additional Details	Action Items	Timeline
	Home or Community	Classroom	School	System	Ministry			
2. A focused plan to build formal leadership capacity in mathematics should be developed, communicated and supported and should include specific supports (eg. Ministry resources and personnel, School Effectiveness Framework, Ontario Leadership Framework and OPC Mathematics webinars) and current research. This plan should include school Administrators, Central Office Staff in Program, Student Success and Special Education as well as Senior Administration.		X	X	X		Leadership capacity in mathematics, whether it be at the school or system level, is integral to creating improvement in mathematics teaching and learning.	Create a committee comprised of Senior Administrators, School Administrators, Ministry Partners and Central Office staff. They will work with the team in Consideration 1 to develop a distinct plan for building the capacity of school leaders to lead their school in the work outlined in the comprehensive math action plan. The monitoring of this work with be linked to Consideration 11.	An initial plan by June 2016 for implementation beginning September 2016. Quarterly meetings to review monitoring data and review progress.

Considerations	Areas of Impact					Additional Details	Action Items	Timeline
	Home or Community	Classroom	School	System	Ministry			
3. Revise the GECDSB Math Vision to include the graphic and description of the three realms (Social & Emotional, Physical, Choice & Voice) of the Responsive Math Learning Environment and reflect the comprehensive plan of Consideration 1 and the monitoring of the implementation of Consideration 11.		X	X	X			Establish a representative group who owns the authorship and sharing of the GECDSB Vision for Mathematics. This should include multiple authors from multiple departments. Revise to include the three realms of a Responsive Math Learning Environment. Revise the Learning Environment graphic to include the inter-relationships of these three realms. Revise where necessary to reflect the work of the Math Task Force.	Begin Immediately for launch of renewed vision September 2016.

Considerations	Areas of Impact					Additional Details	Action Items	Timeline
	Home or Community	Classroom	School	System	Ministry			
4. A student-centered learning community (District Review) should be developed and implemented. This process should; a. invite educators to be host teachers and engage in the "Classroom Visits" protocol around the Responsive Mathematics Learning Environment and; b. invite other stakeholders to visit and join the debrief in person or through live streaming.	X	X	X	X		1. Engage teachers and students in observations and debrief. 2. Pilot the Student-Centered Learning Communities (District Review) with one school and look to see how this protocol could inform the SIPSA. 3. Start to implement the Student-Centered Learning Communities (District Review) process by selecting schools that expressed interest in Instructional Rounds and who have high intensity support through the Program Department.	1. Engage teachers and students in observations and debrief. 2. Pilot the Student-Centered Learning Communities (District Review) with one school and look to see how this protocol could inform the SIPSA. 3. Start to implement the Student-Centered Learning Communities (District Review) process by selecting schools that expressed interest in Instructional Rounds and who have high intensity support through the Program Department.	Begin immediately for full implementation September 2016

Considerations	Areas of Impact					Additional Details	Action Items	Timeline
	Home or Community	Classroom	School	System	Ministry			
5. Develop a system-wide understanding of "teacher expertise" in regards to mathematics, and focus the learning identified in Consideration 1 on building teacher expertise for all teachers in the area of mathematics.		X	X	X		We believe that all educators can teach to high standards given sufficient time and support. We will define what teacher expertise and high quality mathematics teaching and learning looks like, and root all learning in these understandings.	This will be part of the work of the committee in Consideration 1, as a foundation to planning for learning in mathematics.	Begin immediately, implementation ongoing

Considerations	Areas of Impact					Additional Details	Action Items	Timeline
	Home or Community	Classroom	School	System	Ministry			
6. a. Educator learning should be supported through the provision of mathematics instructional coaches in the elementary system. These supports should include: comprehensive professional learning models (Consideration 1), support and development of leadership capacity (Consideration 2) and should be sustained until the point at which the monitoring of the GECDSB Math Vision implementation (see Consideration 11) suggests deep implementation has been achieved in all classrooms for all students. As well, a model of professional learning for mathematics coaches should be developed (Consideration 2). b. Alignment of the existing coaching model in secondary schools should be based on school need in order to support teaching and learning as outlined in Consideration 1 and the GECDSB Vision for Mathematics.		X	X	X		There are a variety of models in place provincially, and experience with effective models locally, that need to be considered. The most effective way to change teacher practice and build leadership capacity is through the provision of math coaches to schools, but they need to be in schools often enough to support ongoing learning and sustained changes in practice.	1. Create a committee to match successful coaching models to GECDSB budgets, in consideration of the pending 2016 EPO and GSN announcements. For secondary coaches, ensure that the development and provision of mathematics supports is available based on school need. 2. Implement the initial model through the 2016 elementary staffing process. 3. Monitor the impact of the coaching model (see Consideration 11). 4. Review the model semi-annually, and commit ongoing financial support to this area.	1. Committee creation by April 2016. 2. Consideration by May 2016. 3. Implementation September 2016 4. Ongoing monitoring and review to inform evolutions in structure for 2017-2018. 5. Repeat annually until deep implementation of GECDSB Math Vision is evident.

Considerations	Areas of Impact					Additional Details	Action Items	Timeline
	Home or Community	Classroom	School	System	Ministry			
7. Specific supports should be provided for teachers in grades 7-9 which focus on co-constructing understanding of effective instructional practices, understandings, and expectations, including consistent assessment practices, and to better support understanding of the development of math concepts throughout the curriculum, student transitions and pathway selections.		X	X	X		This work is currently supported under the "Middle Years Collaborative Inquiry" (MYCI) work. Should funding for this work be realigned provincially, a commitment should be made that the elements valued in this Consideration are supported through new structure or incorporated into Consideration 1.	1. Establish a collaborative working group consisting of Program, Student Success and Special Education. 2. Ensure that the model of MYCI supports the Consideration given, or, should 2016 Ministry funding announcement determine otherwise, ensure that this work is incorporated into Consideration 1. 3. This working group should work with the monitoring group in Consideration 11 to reflect and adapt on this work and to inform future changes and collaborations.	1. Begin immediately as part of existing MYCI 2. Quarterly review of monitoring data to reflect and adapt programming to meet the stated goals. 3. Annual considerations made based on funding to ensure these priorities are owned across departments and reflected in the provision of these supports.

Considerations	Areas of Impact: Home or Community	Classroom	School	System	Ministry	Additional Details	Action Items	Timeline
8. Classroom timetables should be parallel to, and be consistent with, Special Education Resource Room (SERR) classroom schedules, so as to ensure the alignment of mathematics instruction. In the case in which, a student with special education needs returns to the homeroom and math instruction continues, accommodations and modifications should be in place to support the individual needs of the student and all students.		X	X	X			1. Establish a representative group to develop a plan of how this work will be piloted and implemented in our schools.	By June 2016, to inform potential changes or pilots commencing September 2016

Considerations	Areas of Impact					Additional Details	Action Items	Timeline
	Home or Community	Classroom	School	System	Ministry			
9. A system-wide model for early identification of students with math learning struggles should be developed. In addition a system-wide response to these learning needs should be developed and include appropriate, effective, specific, and timely interventions that support student learning.		X	X	X		This will also include considering the development of a Early Years/Primary math diagnostic tool to identify gaps in student understanding and inform system, school, and classroom level responses.	1. Establish a committee comprised of teachers, School Administrators, Program and Special Education staff, and Senior Administration. 2. This committee will work collaboratively to develop the model required by this consideration. 3. This committee will work with the monitoring group established through Consideration 11 to measure the impact of this work, and to reflect upon and adapt approaches and resources as necessary.	Begin immediately, and with urgency, for implementation at earliest possible time.

Considerations	Areas of Impact					Additional Details	Action Items	Timeline
	Home or Community	Classroom	School	System	Ministry			
10. As part of a comprehensive plan, specific consideration should be given to the supports available for student populations with historic gaps in achievement including: students with special education needs, students who are from First Nation, Metis, and Inuit backgrounds, students at risk/in risk and English Language Learners (ELL) especially ELLs with limited prior schooling. This plan should also give consideration to developing understanding and responses to other learning barriers such as socio-economic status and lack of familial supports.		X	X	X		The same group as Consideration 1 may consider this part of their work. Otherwise, a separate working group will be established.	1. Determine whether this Consideration can be combined with Consideration 1. 2. Work collaboratively with School Administration, Central Office Staff, and Senior Administration to develop and implement this plan. 3. This committee will work with the monitoring group established through Consideration 11 to measure the impact of this work, and to reflect upon and adapt approaches and resources as necessary.	Already underway, more formal and comprehensive plan for September 2016. Implementation and monitoring ongoing.

Considerations	Areas of Impact					Additional Details	Action Items	Timeline
	Home or Community	Classroom	School	System	Ministry			
11. A comprehensive, long-term model for data collection and monitoring should be developed, which allows for the system to be responsive to math teaching and learning. The model should be a collaborative, cyclical model in which the values, opinions, beliefs, perspectives, and cultural background of stakeholders (students, all educators, parent/guardian, special interest groups, workplace and post-secondary institutions, external experts and community partners) will be considered, included, valued, listened to, and acted upon. This data collection and monitoring should be transparent and shared with all stakeholders and be used to inform system practices and policy in order to support teaching and learning. Additional consideration should be given to a continuation of the existing Math Task Force to support monitoring. This group would meet regularly and be responsible for bi-annual reports presented to the Board of Trustees, Senior Administration, GECPIC, SEAC, IPC, Student Senate and other staff.		X	X	X	X	This consideration is an essential component of the overall plan and the group will be responsible for the ongoing, sustained, and focused monitoring of the consideration implementations and impacts, and will support each working group by providing data and analysis to inform future directions.	1. Data collection and monitoring committee established and roles defined. 2. Develop data collection and monitoring plan. 3. Establish the composition of an ongoing Math Task Force, including meeting frequency and reporting responsibilities. 4. Connect with other working groups established by this report to inform the ongoing data plan, create a schedule for reflection and analysis meetings, and establish reporting timelines to specific departments, Senior Administration, Trustees, and the community.	Plan by June 2016, implementation September 2016. Extended Math Task Force confirmed by June 2016

Considerations	Areas of Impact					Additional Details	Action Items	Timeline
	Home or Community	Classroom	School	System	Ministry			
12. An exploration of the foci and prioritization of existing system practices, supports and department structures, inclusive of how they work, should be done in order to determine whether they are most effective in providing comprehensive and differentiated professional learning as outlined in Consideration 1. All departments should refine their work based on the GECDSB Math Vision and the plan for professional learning.		X	X	X		The work within this Consideration should be in conjunction with the monitoring team established in Consideration 11 to provide ongoing, data-informed, reflective evolutions of practice and policy. It should also include a review of current hiring and staffing practices to consider how they support the math action plan and how they do, or could, support improved math outcomes for students. Considerations should also be given to existing practices, structures and policies and how they may be adapted to better meet student needs. This includes, but is not limited to, time allocation to teaching mathematics (including the integration of math across the curriculum)Learning Commons, Learning Support Teachers, Student Success Teachers, the role of Primary prep coverage, and the benefits of a non-semestered approach to mathematics in the secondary panel.	1. Each department conduct a review of how they support teaching and learning in mathematics, including the learning environment and provision of resources and supports. This will include, but is not limited to, reviewing existing roles in the school, as well as current structures and policies impacting the learning environment (e.g. Internet filtering, technology provision and support, purchasing procedures, etc.). 2. Consideration should also be given to non-semestered math programming in Grade 9 and 10	June 2016, immediate implementation where appropriate, September 2016 for school-based changes, with any pilot projects occurring in 2016-2017 for potential implementation September 2017.

Considerations	Areas of Impact					Additional Details	Action Items	Timeline
	Home or Community	Classroom	School	System	Ministry			
13. Resources should be developed that support families in promoting and facilitating math learning at home and should be connected to appropriate grade level expectations. These should include access to Ministry of Education supports (e.g. Parent Guides) and other services (e.g. Homework Help).	X	X	X	X	X	Supporting families is an essential component of promoting the importance of numeracy. We need to connect parents to the mathematics in their child's class and provide them with sufficient resources and supports to extend math learning beyond the classroom.	1. Establish a group responsible for the development of new resources and the organization of existing resources. 2. Create an area on the publicboard.ca website specifically for parent and home support and resources for mathematics and see that it is updated and maintained on a regular basis. 3. Develop a communications plan that includes administrators, educators and parent councils to share and support the use of these materials linked to the learning in their schools.	Develop plan by June 2016. Existing resources organized and posted by September 2016 with board-specific resources and information added on an ongoing basis.

Considerations	Areas of Impact					Additional Details	Action Items	Timeline
	Home or Community	Classroom	School	System	Ministry			
14. Opportunities for partnership with various stakeholders should be explored and leveraged under the premise of supporting teaching and learning and creating conditions for knowledge mobilization both locally and globally. These partnerships should develop and promote models of reciprocity that support teaching and learning.	X	X	X	X	X		**Locally:** (teachers, parents, students, employers, local sector leaders) Skills Canada Indigenous Parent Committee Greater Essex County Parent Involvement Committee Special Education Advisory Committee Student Senate Diversity Office Faculty of Education University of Windsor (math department) Coterminous boards St. Clair College City of Windsor Partner with Parent and Family Literacy Centres, Ontario Early Years Centres, and Daycare Providers to support their understanding of initial math learning for pre-school students, based on the work of Dr. Cathy Bruce. **Provincially** Ontario College of Teachers Ministry of Education People for Education Learning Disabilities Association of Ontario Chiefs of Ontario Aboriginal Education Office Ontario Association of Mathematics Educator Education Quality and Accountability Office Ontario Institute for Studies in Education Trent University (Dr. Cathy Bruce) **Globally** Reciprocal Learning Sister School Committee Program for International Student Assessment United Nations National Council of Teachers of Mathematics International Congress for School Effectiveness and Improvement Write an end of the year report which shares the ways in which GECDSB engages in reciprocal partnerships, and the outcomes of these partnerships locally, provincially and globally in relation to mathematics teaching and learning.	1. September 2016 - Establish Partnerships or Council to allow for the gathering and review of information 2. Throughout the year designated staff will make purposeful connections to partner with these stakeholders in relation to mathematics. 3. May-Each designated staff member will provide a brief summary of the math learning and outcomes of their partnerships. 4. Continue this process annually

CONSIDERATION RATIONALE

Consideration 1

A district-wide model for professional learning should be developed, communicated, supported, and celebrated. This model should be multi-year and focused on content learning, pedagogical learning, and pedagogical-content learning. The model should focus on deepening proficiency as defined in the GECDSB Vision for Mathematics and be differentiated according to the needs of schools, departments and educators. The model should serve the goal of improving math learning for every student in every classroom.

In September 2010, the Ontario Ministry of Education brought together a Mathematics Teaching and Learning Working Group in order to “identify what it would take to bring greater depth and coherence to the K–12 mathematics program” (p. 1). This group produced the *Seven Foundational Principles for Improvement in Mathematics, K–12*, which is a guide for schools and school systems. Each of the principles is imperative in guiding effective school and system practices. They serve as a comprehensive framework for system and school planning of mathematics teaching and learning. The fourth foundational principle highlights the importance of schools and schools systems “support(ing) collaborative professional learning in mathematics” (p. 7).

In order to realize the full implementation of the GECDSB Vision for Mathematics, a comprehensive, wide-ranging plan for educator learning must be developed. Based on the work of Schulman (1987), there are three distinct but interrelated domains for educator knowledge that impact the quality of teaching and learning in the classroom. Shulman describes content as the “what” and pedagogy as the “how” of teaching. According to his work, pedagogical content knowledge is a highly specialized skill-set that differs from content specific knowledge and general pedagogical knowledge (Shulman, 1987). As Ball, Hill and Bass (2005) share, the knowledge needed to do mathematics is different from the knowledge needed to teach mathematics.

Within the context of mathematics teaching these include educator’s mathematics content knowledge (what is known and understood about mathematics), pedagogical knowledge (what is known and understood about teaching and learning), and pedagogical-content knowledge (what is known and understood about teaching and learning, specific to the context of mathematics).

Student proficiency is persistently tied to educator proficiency. With the understanding that a teacher is the key to a highly effective math program (Bruce & Flynn, 2013), attention must be given to what professional learning educators might need to develop and support their mathematical teaching. The authors of *Adding it Up* state:

> *Effective programs of teacher preparation and professional development help teachers understand the mathematics they teach, how their students learn that mathematics, and how to facilitate that learning. In these programs, teachers are not given prescriptions for practice or readymade solutions to teaching problems. Instead, they adapt what they are learning to deal with problems that arise in their own teaching. (National Research Council, 2001, p. 10)*

Consideration 2
A focused plan to build formal leadership capacity in mathematics should be developed, communicated and supported, and should include specific supports (e.g. Ministry resources and personnel, School Effectiveness Framework, Ontario Leadership Framework and OPC Mathematics webinars) and current research. This plan should include school administrators, Central Office Staff in Program, Student Success and Special Education as well as Senior Administration.

The *Seven Foundational Principles for Improvement in Mathematics, K–12*, identifies that “All system, board and school leaders commit to providing the resources that support the most effective teaching and learning of mathematics for all students” (p. 5). The ability of the school-based administrator to assume the role of instructional leader and support learning is a significant determining factor in the success of any educational advancement (Leithwood, 1992). Instructional leadership is not limited to school-based administrators, but encompasses the range of personnel tasked with supporting teaching and learning within a school-system. With this said, the Ontario Ministry of Education, Mathematics Teaching and Learning Working Group identifies that there should be very specific and thoughtful planning of leadership capacity specific to mathematics (2010, p.5).

Consideration 3
Revise the GECDSB Math Vision to include the graphic and description of the three realms (Social & Emotional, Physical, Choice & Voice) of the Responsive Math Learning Environment and reflect the comprehensive plan of Consideration 1 and the monitoring of the implementation of Consideration 11.

The learning environment, as suggested by many education researchers (LNS Monograph 27, 2012), is “the third teacher,” and can either enhance or hinder the kind of learning that impacts students’ potential to respond creatively and meaningfully to future challenges. Fraser (2012) identifies that “A classroom that is functioning successfully as a third teacher will be responsive to the children’s interests, provide opportunities for children to make their thinking visible and then foster further learning and engagement” (p.67).

A responsive learning environment is identified as one of the *Seven Foundational Principles for Improvement in Mathematics, K–12* (2010, p. 8). The document recognizes that the “learning environment in all classrooms reflects the commitment of the teacher, the school and the board to meeting the needs of all students in the teaching and learning of mathematics” (2010, p. 8).

Based on the literature, it can be surmised that a responsive learning environment includes three realms: the Physical Realm, the Social and Emotional Realm, and the Choice and Voice Realm.

When considering the Physical Realm, educators look at the space of the classroom that promotes collaboration through group work as well as the space that permits quiet thinking and exploring of math. Active areas for inquiry, investigation and wonder are also considered to be part of the Physical Realm. When students are involved in the process of creating their own learning environment they can develop a sense of community and increased motivation.

When educators work to foster the Social and Emotional Realm, students feel safer to take risks in math class so that they can make mistakes while trying new ideas and strategies. Students also feel safer to revise their ideas and develop new mathematical understandings. When students feel supported by educators, they develop a more positive attitude towards math. Students feel they learn better in a "togetherness" learning environment that provides them with a sense of community. Furthermore, when students have more opportunities to independently explore and inquire, they feel more connected to the classroom community. In these classrooms, students not only hold more positive attitudes toward math, but they demonstrate higher achievement in math (Yang, 2015).

Students need to feel they have a “choice and voice” in their learning. The Choice and Voice Realm is key in creating a responsive math learning environment that encourages diverse thinking perspectives and ideas that are valued as ways to deepen mathematical understanding. Students have choice in exploring mathematics by choosing tasks, tools, methods, and partners. Students need to engage in tasks that challenge their current understandings and therefore have multiple entry points to meet their needs (Suurtamm, Quigly, & Lazarus, 2014; Boaler, 2015).

An effective mathematics learning environment is determined by many factors and is influenced by all levels of educational organizations. Thus, decisions at all levels and by all stakeholders must take heed of the impact of these on the learning environment. For example, the choice of materials and resources to support teaching and learning, such as math kits and technology, would impact the learning environment.

Consideration 4
A student-centred learning community (District Review Process) should be developed and implemented. This process should:
a. invite educators to be host teachers and engage in the “Classroom Visits” protocol around the Responsive Mathematics Learning Environment and;
b. invite other stakeholders to visit and join the debrief in person or through live streaming.

The 2013 School Effectiveness Framework outlines the process for school self-assessment and district review. There is enormous value in educators visiting the classrooms of other educators in order to engage in reflective conversations about student learning. The true power of these models has been evident when teachers are involved in the visitations, rather than it being a measure of external accountability.

Consideration 5
Develop a system-wide understanding of “teacher expertise” with respect to mathematics, and focus the learning identified in Consideration 1 and on building teacher expertise for all educators in the area of mathematics.

The National Council of Teachers of Mathematics (NCTM, 2000) has cited that teacher knowledge, understanding and skill are of central importance in the teaching of mathematics. Thus, content expertise is a critical issue, but the concept of mathematics teaching expertise requires careful study, and the research clearly identifies many significant classroom, school and system considerations.

Students engage in mathematics learning throughout their years of elementary and secondary school. Effectively supporting this learning requires *all* teachers to continue to develop and refine their expertise in mathematics. It is critical for our schools and school system to support deeper and broader understanding of mathematics teaching and learning for all educators. In order to best serve the interests of our students we need to expand our definition of expertise to include content, pedagogical and pedagogical-content-knowledge (Shulman, 1987). This model should be predicated on a comprehensive and research-based definition of proficiency, and include all the domains of expertise (National Research Council, 2001).

Consideration 6

a. **Educator learning should be supported through the provision of mathematics instructional coaches in the elementary system. These supports should include: comprehensive professional learning models (Consideration 1), support and development of leadership capacity (Consideration 2) and should be sustained until the point at which the monitoring of the GECDSB Math Vision implementation (see Consideration 11) suggests deep implementation has been achieved in all classrooms for all students. As well, a model of professional learning for mathematics coaches should be developed (Consideration 2).**

b. **Alignment of the existing coaching model in secondary school be based on school need in order to support teaching and learning as outlined in Consideration 1 and the GECDSB Vision for Mathematics.**

Research has suggested that school-based mathematics coaches may be a vehicle to support the improvement of mathematics teaching and learning in elementary schools (National Research Council, 2001). The intent is to target educators' understanding and action, through partnerships with a knowledgeable colleague who possesses a deep understanding of mathematics pedagogical-content knowledge. These mathematical coaches would serve as an on-site resource for teachers to build collaborative professional development addressing mathematical content, pedagogy, and curriculum in an effort to enhance instruction and improve student achievement (Marzano, Walters, & McNulty, 2005). They would support deprivatizing mathematical practice within schools through developing and supporting the conditions that would allow educators to collaborate with each other in order to build mathematical capacity (Neufeld & Roper, 2003).

Bruce and Ross (2008), in a mathematics peer coaching study, found that when an educator receives positive and constructive feedback from a respected peer, there is even greater potential for enhanced goal setting, motivation to take risks, and implementation of challenging teaching strategies.

Sustained investment in mathematic coaches is crucial (Campbell & Malkus, 2011). Support for mathematics coaches includes a high degree of professional learning addressing mathematics content, pedagogy, and coaching prior to and during at least their first year in the coaching role. Over time, significant positive effects on student achievement can be attained as knowledgeable coaches gain experience and as educators learn and work together.

Consideration 7

Specific supports should be provided for teachers in grades 7-9 which focus on co-constructing understanding of effective instructional practices, understandings, and expectations, including consistent assessment practices, and to better support understanding of the development of math concepts throughout the curriculum, student transitions and pathway selections.

The School Effectiveness Framework K-12 (2013) outlines the areas which define an optimal school system, from the district level to the student desk. Within this there is the aim of consistency, of practice and learning experience, for all students across all grades. One of the principles of the K-12 framework is that factors which influence student learning are similar across all grades. Thus there is great value in engaging in work that supports this consistency.

Creating Pathways to Success (2013) notes that:

> *The transition from elementary to secondary school is among the most challenging periods of adolescence. This transition is a complex process, characterized by a constant tension between "being and becoming" (Tilleczek, 2010a). At this stage of their lives, students are struggling to establish their identities every day, through interactions with friends, at school, at home, and in the community. Immersed in uncertainty on several fronts, they need to feel safe and to experience a sense of belonging. They also need to take courses that align with their strengths, interests, and aspirations (Tilleczek, 2010b).*

In order to facilitate the transition between elementary and secondary, to ensure consistently applied teaching, learning, and assessment practices, and to support the course selection most suited to each individual and their aspirations, there is a need to work collaboratively to developed shared understandings and new learning.

Consideration 8

Classroom timetables should be parallel to, and consistent with, Special Education Resource Room (SERR) classroom schedules, so as to ensure the alignment of mathematics instruction. In the case where a student with special education needs returns to the homeroom and math instruction continues, accommodations and modifications should be in place to support the individual needs of the student.

Currently there is a model for Special Education whereby resource room placements are provided to ensure students with specific, identified needs are given the focused and explicit supports they need to achieve success in school. The students are in these placements for half of the day to receive literacy and numeracy instruction, and in their "homeroom" for the other half of the day. There is a logistical challenge posed to homerooms extending the math block beyond 50 minutes, as students working in the resource room will return prior to the end of the extended math block. It is imperative that these students continue to receive high quality instruction and are integrated into the learning in their class. This is also a vital consideration for creating the opportunity for cross-curricular math learning.

Universal Design for Learning and Differentiated Instruction are familiar principles described in *Learning for All, K-12* (2013). In order to ensure that appropriate accommodations and modifications are in place to enable students to access the learning across curriculum areas and with approaches consistent with those in the Individual Education Plan, these principles need to be consistently applied in all classrooms.

Consideration 9

A system-wide model for early identification of students with math learning struggles should be developed. In addition, a system-wide response to these learning needs should be developed and include appropriate, effective, specific, and timely interventions that support student learning.

Early identification of learning struggles and the provision of appropriate interventions are essential to create the learning foundation necessary for future success of all students (Fullan, Hill & Crevola, 2006). A model is needed that allows identification of, and response to, challenges in student math learning at the earliest possible time, in order to provide every student the best opportunity for future success. The approaches outlined in *Learning for All, K–12* (2013), including Universal Design for Learning and Differentiating Instruction:

> *are designed to bring about personalization and precision in learning, starting from the premise that (1) teachers need to know their students, and (2) assessment for learning, in conjunction with professional learning, is critical to achieving that goal. These approaches provide a road map to assist educators in reaching every student. (p.53)*

Early intervention is particularly important in mathematics as Claessens, Duncan and Engel (2009), Geary, Hoard, Nugent and Bailey (2013), and Ritchie and Bates (2013) all share that early math skills are the best predictor of school and career success, and math is a better predictor than early reading for future language skills.

Low socio-economic status students come to school behind their higher socio-economic status student peers (Starkey, Klein, & Wakeley, 2004; Baroody, 2006) and over time, gaps widen in the absence of intervention (Cannon, Jackowitz & Painter, 2006; Crosnoe & Cooper, 2010).

Consideration 10
As part of a comprehensive plan, specific consideration should be given to the supports available for student populations with historic gaps in achievement including: students with special education needs, students who are from First Nation, Metis, and Inuit backgrounds, students at risk/in risk and English Language Learners (ELL) especially ELLs with limited prior schooling. This plan should also give consideration to developing understanding and responses to other learning barriers such as socio-economic status and lack of familial supports.

In addition to providing early interventions for students with mathematical struggles, there is a need to address student populations with historic challenges and gaps in achievement. "Achievement gap" is a widely used term which refers to the discrepancy in academic achievement of identifiable groups of students. Learning for All: A Guide to Effective Assessment and Instruction for All Students (2013) is an Ontario Ministry of Education publication which guides and supports school and system planning by addressing effective means of reaching all learners. It states:

> *Gaps in achievement can be measured in terms of various factors, such as gender, ethnocultural background, socio-economic status, special education needs, language proficiency, or number of credits accumulated by the end of a particular grade. Achievement gaps can also be defined according to combinations of these factors, such as gender and special education needs, or gender and socio-economic status, or ethnocultural background and credit accumulation by year and grade. (p. 11)*

Better understanding of the learning experience of these students is essential to providing the programming and supports that will allow them to be successful in their learning. Once these factors are identified and understood, they need to be addressed as part of an overall board vision and approach, across subject areas and panel.

Effective mathematics teaching, rooted in Universal Design for Learning, supports learning according to the differentiated needs of the student. In order to overcome the disparities in achievement, we must address learning gaps, which is a term "often used to refer to the gap between a student's actual achievement and his or her potential for achievement" (Learning for All, 2013). Our responsibility is to address these learning gaps with precision and intentionality.

Consideration 11

A comprehensive, long-term model for data collection and monitoring, which allows for the system to be responsive to math teaching and learning, should be developed. The model should be a collaborative, cyclical model in which the values, opinions, beliefs, perspectives, and cultural background of stakeholders (students, all educators, parent/guardian, special interest groups, workplace and post-secondary institutions, external experts and community partners) will be considered, included, valued, listened to, and acted upon. This data collection and monitoring should be transparent and shared with all stakeholders and be used to inform system practices and policy in order to support teaching and learning. Additional consideration should be given to a continuation of the existing Math Task Force to support monitoring. This group would meet regularly and be responsible for bi-annual reports presented to the Board of Trustees, Senior Administration, GECPIC, SEAC, IPC, Student Senate, and other staff.

The work of Hattie (2009) is very clear in the need for educators and education systems to know their impact. Decision making that is informed by relevant local data and global education research is a foundation for progressive, responsive school systems to support the learning and needs of the schools they govern. To do this, a renewed commitment to refined, focused, ongoing data collection, monitoring, reflection, and consultation is essential. This information then needs to be acted upon through policy and program approaches that respond to the evident needs and that develop the existing strengths recognized in the work of educators. This work also creates a transparent operational approach and a sense of collective responsibility towards the learning of the students in our care.

Consideration 12

An exploration of the foci and prioritization of existing system practices, supports and department structures, inclusive of how they work, should be done in order to determine whether they are most effective in providing comprehensive and differentiated professional learning as outlined in Consideration 1. All departments should refine their work based on the GECDSB Math Vision and the plan for professional learning.

As the work of Leithwood (2013) and the Ontario Leadership Framework (2013) show, it is essential that a clear vision be established if improvement is to be realized. Within the education context, this vision needs to be understood and supported across the organization. Significant and sustained improvements in mathematics outcomes for our students requires a collaborative and comprehensive approach that includes both academic and non-academic departments reflecting on their role in, and contribution to, mathematics learning in the system.

Consideration 13

Resources should be developed that support families in promoting and facilitating math learning at home and should be connected to appropriate grade level expectations. These should include access to Ministry of Education supports (e.g. Parent Guides) and other services (e.g. Homework Help).

Parental engagement has been identified as a mitigating difference in socioeconomic status and student achievement (Fan & Chen, 2001; Henderson & Mapp, 2001; Bolivar & Chrispeels, 2011; Ma, She & Krenn, 2013). Although the literature typically uses the term *parent*, the term *family* is used to encourage inclusiveness of all contexts. Family engagement can and does make a positive difference for students.

In a report commissioned by the Council of Directors of Education and the Institute of Education Leadership entitled *Strong Districts and Their Leadership,* Kenneth Leithwood (2013) identifies a productive relationship with staff and other stakeholders as one of the critical features of strong school districts. The research goes on to describe how within these contexts, there is an emphasis on joint responsibility for working toward success of the common goal of improved student achievement (Leithwood, 2013). In addition, the district imperatively builds relationships with the community in order to accomplish their mission and goal. This connects to the primary and critical feature of strong districts, which is a broadly shared mission, vision and goals founded on ambitious images of the educated person.

Specific to the notion of family engagement and support for mathematics, Vukovic, Roberts, & Wright (2013) state that policies and programs targeting involvement in mathematics should focus on home-based practices that do not necessarily require technical mathematical skills. Further noted is the idea that parents and families should receive training, resources and support on culturally appropriate ways to create home learning environments that foster high expectations for their children's success in mathematics. This research holds true for elementary students, but is also applicable to secondary students. Jeynes (2007) specifically focused on the achievement of secondary school students. One of the patterns that emerged from the findings was that subtle aspects of parental involvement, such as parental style and expectations, had a greater impact on student achievement than more demonstrative factors like household rules and parent participation in school functions.

Consideration 14

Opportunities for partnership with various stakeholders should be explored and leveraged under the premise of supporting teaching and learning and creating conditions for knowledge mobilization both locally and globally. These partnerships should develop and promote models of reciprocity that support teaching and learning.

Mathematics achievement is a global focus and information shared among various groups serves to enhance the learning of the collective. Partnerships like those with local and global associates are also important in leveraging and mobilizing knowledge to improve student outcomes. The Reciprocal Learning Program is one example of global partnership between the University of Windsor, the Greater Essex County District School Board, Southwest University, and Chong Qing schools, which serves to enhance learning for all participating parties (Xu & Connelly, 2013).

Partnerships such as these promote cross-cultural perspectives and new approaches to research on curriculum and their application in classrooms. The goals of the Reciprocal Learning Program are: to provide an exceptional learning experience for teacher candidates, educators and administrators; to expand perspectives regarding societies of increased diversity; to foster international collaboration among faculty members who are interested in cross-cultural studies; to promote multicultural education; and to enhance international education relationships (Xu, 2011).

Schools and school systems benefit from rich, responsive and reciprocal relationships with local and global partners. Practices that support structures, systems and professional learning tied to the engagement of these partners will serve to enhance the experience of all stakeholders and ultimately benefit students.

CONCLUSION

The purpose of this document is to provide considerations as to how best support future planning in the area of mathematics teaching and learning. This report is grounded in the principles of the Full-Day Early Learning—Kindergarten program and the Ontario Mathematics Curricula for grades 1-8, 9-10, and 11 & 12 as well as the adopted core beliefs of the GECDSB. It includes a review of current research as well as the voices of GECDSB stakeholders. It serves as a source for innovative long-term planning.

Improvement in student achievement comes as a result of focused and intensive learning across a school-district. We urge the GECDSB to proceed with actions and supports that are committed to improving student outcomes in mathematics. This requires everyone in the organization, regardless of position or role, to mobilization resources and actions under the premise of "all hands on deck". Mathematics is a universal human endeavor. It is a rich and diverse discipline. The goal of mathematics education should comprise this depth and richness.

The goal of mathematics proficiency for every Greater Essex County District School Board student is both ambitious and necessary. We educate for excellence. We must extend our discourse in mathematics education to honour the essence of mathematics because in every classroom sit artists, writers, builders, scientists and mathematicians. It is the fullness of the educational experience that reaches the heart of every learner. Our purpose as educators is not to count the limited prospects but to inspire the infinite possibilities.

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Appendix A: A Vision for Mathematics

The GECDSB provides mathematics education that engages and empowers students through collaboration, communication, inquiry, critical thinking and problem-solving, to support each student's learning and nurture a positive attitude towards mathematics.

Whenever we strive to improve in any way, a vision of what that improvement might be is essential. We need to know what we are working towards and why that is important. We also need to know how we plan to reach that vision.

This vision has been developed specifically by and for the Greater Essex County District School Board through consultations with a wide variety of stakeholders including elementary and secondary teachers and administrators, program staff, Student Success, and Special Education. The intent of this vision, and the related strategies and approaches to mathematics teaching and learning, is to support schools and educators as they reflect on the needs of their students and how they will address them as part of their ongoing School Improvement Plans.

Within this vision there are various responsibilities we assume. As a school board, we believe our responsibilities are to create conditions for mathematics learning:

- where competent and knowledgeable educators integrate instruction and assessment;
- where educators and administrators are committed to ongoing learning about mathematics and mathematics instruction;
- where learning environments nurture positive attitudes towards mathematics; and
- where all students have opportunities and support to learn significant mathematics with depth and understanding.

It is the belief of the board that where this vision is actively pursued, and where these responsibilities are met, student achievement in mathematics will increase.

This document will outline some of the strategies, approaches, theories, supports and resources that should be used to meet this vision and these responsibilities.

A Picture of Mathematical Literacy

What is "Mathematical Literacy"?

Conceptual Understanding is the ability to understand mathematical concepts, operations, and relationships.
Procedural Fluency is the skill in carrying out procedures flexibly, accurately, and efficiently, and knowing when the procedures should be applied.
Adaptive Reasoning is the capacity for logical thought, reflection, explanation, and justification.
Strategic Competence is the ability to formulate, represent and solve mathematical problems using an effective strategy.
Productive Disposition is the inclination to see mathematics as useful and valuable.

In order to begin any conversation around improving mathematics we need to share a common understanding of mathematical literacy.

> ***Conceptual understanding*** *is knowledge about the relationships or foundational ideas of a topic.* ***Procedural understanding*** *is knowledge of the rules and procedures used in carrying out mathematical processes and also the symbolism used to represent mathematics. (van der Walle , Karp, Bay-Williams, 2010)*

An example of this is in the task 55 x 24. The *conceptual understanding* of this problem includes the idea that the problem could be represented as repeated addition, and that the problem could be represented in terms of the area of a quadrilateral, the number of seats in a theatre, and any other scenario they can conceive. The p*rocedural knowledge* could include the ability to carry out the standard algorithm (multiply 55 by 24). The ability to go beyond this algorithm or to create an algorithm (for example, 50 x 20, plus 5 x 20, plus 50 x 4, plus 5 x 4) requires *conceptual understanding* of place value and multiplication.

The Ontario Curriculum refers to ***adaptive reasoning*** when,

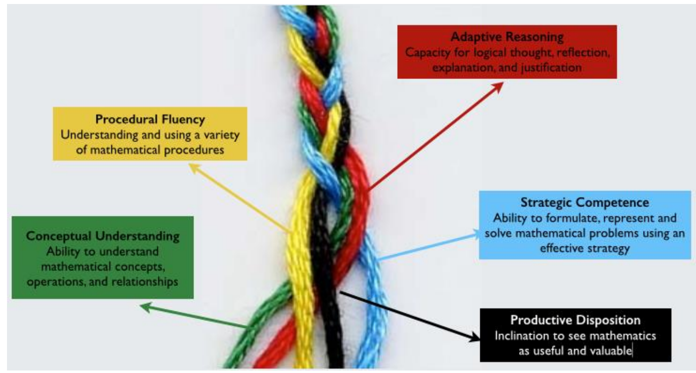
> teachers help students revisit conjectures that they have found to be true in one context to see if they are always true. For example, when teaching students in the junior grades about decimals, teachers may guide students to revisit the conjecture that multiplication always makes things bigger. (The Ontario Curriculum Grades 1-8, Mathematics, 2005, p.14)

It is tempting for teachers to define terms or provide explanations for formulas at the point where students are making conjectures. "Gallery walks and math congresses are opportunities to treat children as developing mathematicians, which emphasizes developing arguments and proofs to convince others." (Models of Intervention in Mathematics)

In approaching a problem, if you feel like you could apply a known or new strategy to solve the problem, try different approaches when the one you selected does not work, and/or create a model to represent your mathematics, this is evidence of ***strategic competence****.*

Mathematically literate people believe they can be successful and are persistent in their approach to problem solving. It is vital that teachers, students, and parents all strive to develop a ***productive disposition*** towards mathematics.

The following image is adapted from *Adding it Up* (National Research Council, 2001) and is used to show that the each of the elements of mathematical literacy are interwoven to reach the goal of being mathematically literate.



While conceptual and procedural understanding of any concept are essential, they are not sufficient. Being mathematically proficient encompasses all five elements of mathematical literacy. While we may place more emphasis on one element than another at any given moment in time, it is the relationships and links between them that underpin mathematical proficiency.

Eight Considerations When Planning for Mathematical Instruction

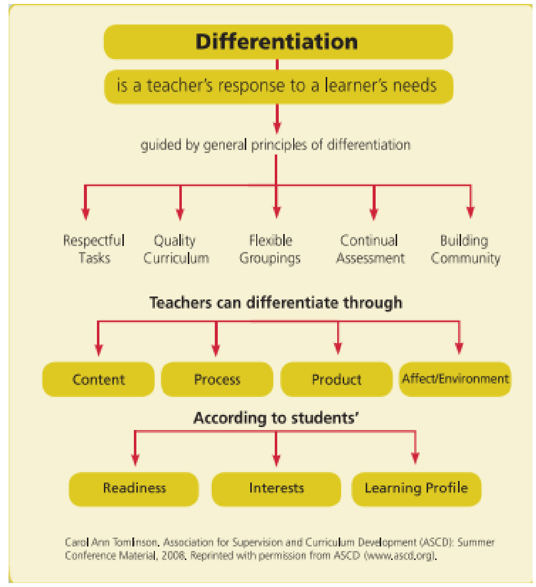
1. Program Scope and Planning
Educators consider curriculum expectations, strands, mathematical processes, and big ideas when planning and using curriculum-appropriate resources.

'By organizing content around big ideas, teachers can teach more efficiently, but most importantly, students can make connections between seemingly disparate topics that help them learn new mathematical ideas.' Marian Small from Making Math Meaningful to Canadian Students, K-8 2013

"Life-long learners of mathematics build new knowledge and skills in prior knowledge using the mathematical processes" From MathGains. For more information about the math processes please go to the following website and select 'Introduction and Overview' http://edugains.ca/newsite/math2/mathematicalprocessesvideo.html

2. Teaching and Learning
Educators focus instruction on providing students opportunities to engage in minds-on tasks, mathematical inquiry, and consolidation of their developing understanding of the big ideas.
Educators consider content, process, product, readiness, interests, the student learning profile, and IEP expectations to effectively differentiate instruction to reach all students. Teachers can empower students to feel that mathematics is something he or she can learn through the use of a variety of lesson styles and by differentiating instruction.

Students vary dramatically in their interests, abilities, learning styles and prior knowledge. Differentiated Instruction based on student learning profiles, allows teachers to respond, through planning, to the needs of the current math learners within their classrooms. Using different assessment strategies, teachers determine where each student is on a landscape of learning or developmental continuum. By also identifying key math concepts and their connections to curriculum expectations, teachers can then meet the mathematical needs of students while varying other aspect of the lessons and tasks students are required to do (based on student interest and/or readiness).



Inquiry based learning in Mathematics creates a balance between learning based on procedure and learning through problem solving and actively engages all students. This teaching strategy naturally lends itself to differentiated instruction and differentiated assessment and when all students have an entry point the outcome is increased student success. Teachers' careful selection of content rich tasks, Minds On activities that active prior knowledge and make students current thinking visible, and the effective use of consolidation, will deepen students understanding of the curriculum and ensure that all students regardless of their ability, move forward.

3. Learning Environment

Educators use appropriate physical classroom arrangements and group students to promote collaboration, communication and a positive, safe learning environment.

4. Student Tasks

Educators provide an appropriate balance of mathematical tasks including the practice of skills, application of procedures, integration of math processes, and rich problem solving. Even if students have not mastered basic skills, they have opportunities to engage in rich tasks that give them a context for these skills.

The tasks that students are asked to do help them to become mathematically literate as outlined in section 2. Tasks should address curriculum and IEP expectations and take into account the readiness, interests and learning styles of the students in the class. Tasks should be derived from multiple sources and resources, and should allow students ample opportunities to collaborate to develop new math knowledge, and communicate their understandings and wonderings about mathematics.

5. Constructing Knowledge

Educators recognize that a balanced approach is the foundation of the mathematics program for all students. Effective questioning activates student's prior knowledge, prompts mathematical thinking, and helps students to construct knowledge. Educators also use a skill-based or conceptual approach when appropriate.

Educators recognize that for students to be mathematically literate, and for them to fully understand the mathematical concepts, they have both conceptual understanding and procedural knowledge, they have the capacity for logical thought, reflection, explanation, and justification (adaptive reasoning), they have the ability to formulate, represent and solve mathematical problems using an effective strategy (strategic competence) and a positive disposition towards mathematics and mathematics learning.

6. Manipulatives and Technology

Educators provide students opportunities to use manipulatives and make use of technology to represent mathematical concepts and procedures, solve problems, and communicate their mathematical thinking and understanding.

Educators understand that manipulatives can support students in developing deeper conceptual understandings but that they must also be able to communicate and understand the math they represent. They also recognize that technology has limited capacity to support actual problem solving, but great scope to support students in communicating their thinking and understanding of mathematics to an audience beyond their classroom. The use of technology can also help students to reflect upon their understandings, learn from the understanding of others (adaptive reasoning), and consider different approaches to solving mathematical problems (strategic competence).

7. Students' Mathematical Communication

Educators provide opportunities for students to use communication as both a way to learn mathematics and a way to articulate ideas. Oral, written and physical communication make mathematical thinking observable.

Mathematical communication is an essential process for learning mathematics because through communication, students reflect upon, clarify and expand their ideas and understanding of the mathematical relationships and mathematical arguments. (Ontario Ministry of Education, 2005)

The Ontario Curriculum (Ontario Ministry of Education, 2005) also emphasizes the significance of communication in mathematics, describing it as a priority of both the elementary school and the secondary school programs. Students communicate to:

- build understanding and consolidate learning;
- ask questions, make conjectures, share ideas, suggest strategies, and explain their reasoning; and
- learning to distinguish between effective and less effective strategies.
- Communication in the math classroom exists in a number of forms, as illustrated in this table.

The use of mathematical language helps students gain insights into their own thinking and develop and express their mathematical ideas and strategies, precisely and coherently, to themselves and to others

Through listening, talking and writing about mathematics, students are prompted to organize, re-organize and consolidate their mathematical thinking and understanding, as well as analyze, evaluate and build on the mathematical thinking and strategies of others.

Communication	Oral	Written	Symbolic, Graphical, or Pictorial	Physical
Types	Teacher-student (or small group) Teacher-whole class Peer talk Self-talk	Personal writing; Descriptive writing; Process Writing; Word problems	Invented symbolism; Pictorial representations; Graphs; Dynamic; Numeric; Algebraic	Concrete actions
Opportunities	Connections to prior learning; Shared problem-solving; Explanations; Predictions; Comparison of ideas and approaches to approaches to problem-solving; Manipulatives; Relationships; Justification	Prior to learning students can what they know about a specific topic; During learning, students can write to clarify their thinking; After learning, students can add new learning their prior learning or summarize their learning	Use mathematical symbols to represent ideas and demonstrate understanding; Data displays, e.g., pictographs, line graphs, scatter-plots, etc. Mathematical modeling using technology, e.g., virtual manipulatives, graphing software, etc.	Use of manipulatives to communicate thinking; Describing or explaining a concept through demonstration, e.g., describe or explain the concept of perimeter concretely by walking the around the entire outside edge of the patio

8. Assessment

Educators assess for different purposes using a variety of assessment strategies and tools. Assessment practices are fair, equitable, and transparent.

The fundamental purpose of assessment and reporting is to improve student learning. The first of the Seven Fundamental Principles of Growing Success; Assessment, Evaluation, and Reporting in Ontario Schools, (2010) emphasizes the need for teachers to use fair, equitable, and transparent assessment and evaluation practices and procedures to support student learning. What does fair, equitable and transparent assessment look like in mathematics?

Fair assessment and evaluation in mathematics involves…

- multiple opportunities for students to demonstrate the full range of their learning in a variety of contexts;
- curriculum expectations and learning goals that relate to the interests, learning styles and preferences, needs and experiences of all students
- multiple opportunities for students to demonstrate their learning, receive descriptive feedback, and time to act upon that feedback prior to assessment of learning; and
- varied and parallel instructional experiences, including the use of manipulatives and technologies

Transparent assessment and evaluation in mathematics involves…

- ongoing descriptive feedback to students that is clear, specific, meaningful and timely to support improved learning and achievement; and
- learning goals shared with students to identify the intended student learning; and

- success criteria that describe what successful demonstration of the learning goal(s) looks like.

Equitable assessment and evaluation in mathematics involves…
- inclusive support for all students, with attention to those with special education needs, those who are learning the language of instruction (English or French) and those who are First Nation, Métis or Inuit;
- a focus on the same knowledge and skills, while differentiating to meet student needs;
- self-assessment opportunities where students assess their own learning, set specific improvement goals and plan next steps for their learning;
- on-going assessment integrated with instruction to permit teachers to monitor student learning to guide the next steps in teaching and learning.

Paying Attention to Mathematics
http://www.edu.gov.on.ca/eng/teachers/studentsuccess/FoundationPrincipals.pdf

Growing Success; Assessment, Evaluation, and Reporting in Ontario Schools (2010)
http://www.edu.gov.on.ca/eng/policyfunding/growsuccess.pdf

GECDSB BELIEFS

The responses to the following questions reflect our current understanding about mathematics instruction and learning

What is the Connection Between Procedural Fluency and Conceptual Understanding?
Understanding the concepts underpinning mathematics requires individual learners to process information, to make sense of it, and to figure out how to apply it. Memorizing procedures on its own does not develop this understanding. This example from "Making Math Meaningful" by Marian Small illustrates this concept:

"A student who fully understands what 3 x 5 means not only realizes that it equals 15, but, at some point, understands all of the following as well:
- It represents the amount in 3 equal groups of 5, no matter what is in the groups;
- It represents the sum of 5 + 5 + 5;
- It represents the area of a rectangle with dimensions 3 and 5;
- It represents the number of combinations of any 3 of one type of item matched with any 5 of another type of item (e.g. 3 shirts and 5 pairs of pants = 15 outfits);
- It represents the result when a rate of 5 is applied 3 times (e.g. going 5 km/h for 3 hours); and
- It is half of 6 x 5, 5 more than 2 x 5, and 5 less than 4 x 5"

To engage students in understanding concepts, teachers provide opportunities to learn through problem solving, to use manipulatives as models, and to engage in math talk where students explain ideas and consider the ideas of others.

Introducing formal procedures or algorithms too quickly limits opportunities for students to fully understand math concepts, however without their introduction at some point in the learning students will not necessarily be able to explore and expand upon the concepts they are considering, nor will they have solid foundations to make judgments over the reasonableness of their answers and efficiency of their methodology. Students who have opportunities to play with invented procedures and consider alternative procedures shared by peers, learning concepts through problem solving will develop the competency to use procedures and algorithms strategically or with procedural fluency, and to judge their own methods against those used traditionally used in mathematics.

Review the previous section on "Mathematical Literacy" to learn more about the relationship between conceptual understanding, procedural knowledge, adaptive reasoning, strategic competence and productive disposition.

What is Math Talk?
A Math-Talk Learning Community is a community where individuals assist one another's learning of mathematics by engaging in meaningful mathematical discourse. (Hufferd-Ackles, Fuson and Sherin 2004 p. 82)

Despite the importance of high quality math talk, left on their own students are not likely to engage in such talk. Teachers play a pivotal role in facilitating these opportunities.

Dr. Catherine D. Bruce, an assistant professor at Trent University in Peterborough, Ontario, and the author of the LNS Monograph Student Interaction in the Math Classroom, identifies five challenges that teachers face when trying to engage students in high quality interactions during math. These are:

- complexities of teaching mathematics in ways they did not experience as students;
- discomfort with their own mathematics knowledge;
- lack of sustained professional development opportunities;
- greater requirement for facilitation skills and attention to classroom dynamics; and
- lack of time, especially in face of curricular demands.

She outlines five strategies for teachers to encourage high-quality interactions, along with evidence for why each is important and how it works:

- The use of rich math tasks;
- Justification of solutions;
- Students questioning one another;
- Use of wait time; and
- Use of guidelines for math-talk.

Whole class discussions can be facilitated using techniques such as Gallery Walk, Math Congress and Bansho. Although there are many similarities and differences in these strategies (which are listed in the "[Communication in the Math Classroom Monographhttp://www.edu.gov.on.ca/eng/literacynumeracy/inspire/research/CBS_Communication_Mathematics.pdf]"), the main purpose for all three is to develop student's communication abilities in math.

Increasing math talk provides students with the opportunity to explain, defend, and justify their mathematical thinking with confidence.

What is the Impact of Teacher Attitudes and Comfort with Mathematics?

"Teachers model and nurture positive attitudes, self-efficacy and engagement in mathematics. As educators gain the mathematical knowledge for teaching, they become more capable –and confident – in helping students extend and formalize their understanding of mathematical concepts. This can contribute to students' development of positive attitudes toward mathematics and an increase in their sense of self-efficacy. Self-efficacy, which is an individual's belief in whether he or she can succeed at a particular activity, plays an integral role in student success. Bruce and Ross discovered that "increases in teacher efficacy led to increases in student efficacy and outcome expectancy and to student achievement" (2010, p. 10). In turn, strong student self-efficacy can contribute to greater enthusiasm and engagement in mathematics" (Ross, 2007, p. 52). Ministry of Education Ontario, Capacity Building Series #22, Maximizing Student Mathematical Learning in the Early Years, September 2011

Teacher attitudes towards math matter. The Mathematics curriculum, Grades 1-8 calls on teachers to bring enthusiasm to the classroom (p.5). It is important to acknowledge that while some teachers are uncomfortable with math, articulating that sentiment with, "I'm not good at math" should carry the same stigma as a claim to being illiterate. It is important for teachers to project a positive attitude about math for students, demonstrating for them that math can be enjoyable and achievable and that developing mathematical literacy is important for living in today's world.

Learning Mathematics for Teaching

In the following video segment, Dr. Deborah Loewenberg-Ball explains that the knowledge that is necessary for math teaching is different from the knowledge necessary for doing mathematics. The good news for teachers who have been uncomfortable with mathematizing themselves is that anticipating student thinking, planning open questions to illicit big ideas, prompting and questioning to support conceptual understanding requires some understanding of math concepts, but more importantly, it requires instructional competency.

Teacher-Efficacy

In this video segment, Dr. Bruce explains how teacher-efficacy is directly connected to student learning and student achievement. She claims that teacher-efficacy is a more reliable predictor of student achievement than socioeconomic status. That is because when teachers believe they are capable of helping students learn mathematics, they persist in supporting students in the classroom, they are not afraid to engage students in rich problems or to take up incorrect responses. “They let learning take place instead of doing a lot of telling.” In turn, this teacher-efficacy translates into student beliefs that they can learn. With this increased student-efficacy, students persist longer with problems. When teachers see students experiencing success with challenging problems, they realize their power to support learning. Teacher-efficacy impacts instructional practice and student-efficacy. Improved instructional practice and improved student-efficacy positively impacts student achievement.

“Remember that how your students feel about mathematics when they begin school in September rests largely on their previous school experiences and the tone at home. But how your students feel about mathematics when they leave your classroom relies on you. It’s important for you to convey, through actions and words, that mathematics is essential in today’s world. Show enthusiasm for math. Tell your students that you appreciate the usefulness of math. Reinforce for them that you value learning math. Of course if your own experience with learning math was difficult and these comments make you inwardly groan, don’t try to fake an attitude of enthusiasm. Skip the commercial, try engaging students in a discussion of math skills that are essential for daily life and let it convey the message about the importance of math.”
Burns & Silbey, 2000, So You Have to Teach Math? p.86

What Could Time for Math Look Like?
As well as having a significant portion of each day dedicated to math instruction and learning, it is imperative that teachers embed mathematics into other subject areas, allowing students to experience the relevance of math, practical and every day applications of math, and the contexts within which math exists in all areas. A comprehensive approach to mathematics would be one where math is evident across the curriculum, time is dedicated for math problem-solving and inquiry, and students are supported in becoming mathematically literate.

Before/minds on	
STUDENT	TEACHER
- Make connections to, and reflect on, prior learning - Share their thinking through discussion in a math talk learning community	-Model a strategy to encourage students to make connections to, and reflect on, prior learning (e.g. Think/Pair/Share, Ticket In, Video Clip, Math Language Recall etc) - Establish expectations and procedures (e.g. roles, groupings, manipulatives, etc) - Activate students’ prior knowledge by posing a thought-provoking question/task that sets the stage for learning - Promote a positive classroom environment using math talk learning communities

The three-part math lesson is one effective component of a comprehensive mathematics program and emphasizes student’s overall conceptual understanding through problem solving, math talk, questioning, and differentiated instruction. The three-part lesson is an inquiry based model where “students are recognized as the ones who are actively creating their own knowledge” (Marian Small).

The following videos are taken from [resources provided by the Literacy and Numeracy Secretariat]() to support instruction in mathematics and outline the key concepts and theories underpinning the three-part lesson approach, as well as breaking each part down to show what the possibilities are.

During/action	
STUDENT	TEACHER
- Flexible grouping; pairs, small groups, or independent - Work to make sense of the problem in their own way to deepen and clarify their thinking - Communicate their thinking to one another and teacher through math talk - Use errors as an opportunity for learning - Make their thinking visible - Make connections to other subjects and real-life contexts	-Provide a problem with multiple points of entry - Group students purposefully - Ask probing questions to help focus students' thinking without leading to strategies or solutions - Encourage accountable math talk - Encourage students to represent and explain their thinking - Reconvene the whole group to answer questions or clarify thinking - Observe and assess

After/consolidation	
STUDENT	TEACHER
- Make connections between mathematical ideas and strategies - Apply descriptive feedback based on learning goals and success criteria - Complete a final practice assessment or reflection to demonstrate consolidated learning	Strategically facilitate whole-class and small-group discussions and sharing by: - Asking questions to clarify misunderstandings - Encourage students to explain and understand a variety of solution strategies without evaluation - Summarizing the discussion and emphasizing key points or concepts (i.e. “naming” the math).

What is the Role of the Textbook?

The Ontario Curriculum dictates what should be taught in all mathematics programs. Historically there has been an over-reliance on the textbook at the expense of a focus on curriculum expectations and opportunities to explore math more deeply, not to mention an oversight in the fact that the textbook and curriculum do not completely align.

The emphasis in mathematics instructions needs to be on delivering the content expected in the curriculum in a way that the student can best learn. The textbook does play a role in this process, but is not the driving force behind instruction and is merely a resource to support the teacher in structuring learning opportunities for students. It is not the textbook itself that is a potential problem, more how it is used and what role it plays in the teaching and learning of mathematics.

The following video from math teacher Dan Meyer outlines some of the ways we need to think more critically about the resources we use to deliver curriculum and what exactly we are asking our students to do in the math lesson.

There are many factors teachers are asked to consider in their use of the textbook. Firstly, they need to ensure that the textbook content aligns directly to the expectations outlined in the Ontario curriculum, and from there make a judgment as to the extent to which these expectations are met. Are supplementary activities required? Does the content of the textbook meet the needs of all learners? Will the students be able to present a clear and coherent understanding of the concepts required by the curriculum through the completion of the tasks in the textbook? Is the textbook the most engaging and/or effective way for the students to learn and demonstrate understanding of these concepts? What other resources might be needed? What other learning opportunities or assessments do the students need?

Once an educator has considered these questions, and acted upon their responses, the textbook will likely have a less prominent role in their math classroom, and other resources and approaches will be in place. The resources will be a useful guide and support for teachers looking to evolve their teaching of mathematics.

What is the Role of the Administrator in Supporting Mathematics?

GECDSB school administrators have a key role to play in improving the mathematics learning in a school. In all areas, not just mathematics, administrators recognize the need to improve as an ongoing process and focus on sustaining those changes that achieve increased student learning, and adjusting the changes that do not. In order to support and promote school improvement, all staff, including administrators learn current theories and best practices as mathematics education quickly evolves around us. Administrators lead all stakeholders in creating and sustaining a positive mathematics culture across the school.

Schools need a shared focused of mathematics learning for all students that has been collaboratively developed and promotes a high level of expectation from the teacher leading to increased achievement. There are going to be multiple barriers to achieving real sustainable growth. By regularly discussing the school's shared focus administrators can encourage optimism in the face of everyday problems.

Through the GECDSB School Improvement Planning process, and the nature of "School-Based Learning", our educators have autonomy and ownership over how and what they learn. The improvement goals that drive this shift need to be identified by all stakeholders. Administrators coordinate professional learning opportunities that value teacher's knowledge and experiences and are based on student learning needs. Administrators work collaboratively with their staff to build a long-term plan, to monitor their incremental growth and to adapt their plan in response to new learning.

How do we Communicate with Parents?

Parents play a vital role in their child's development of mathematics. It is essential that teachers build effective communication between home and school. Teachers are encouraged to articulate with parents the importance of:

- Building strong, positive attitudes about math;
- Beginning with activities that meet your child's level of mathematical understanding;
- Using their first language to explore mathematics at home if they or their child are more comfortable in that language.

Communication is rarely a discrete, individual act but rather occurs within the context of ongoing exchanges (Adler & Rod- man, 1994). Currently, a number of communication opportunities are available to teachers, ranging from blogs, school-to-home communication books, to face to face parent conferences. Every communication exchange, regardless of format, should reflect a thoughtful, planned approach and should be viewed as an opportunity for teachers to promote parent partnerships and, ultimately, to support student learning. Teachers are encouraged to use a variety of strategies, keeping in mind that the more proactive you are the better. The more you explain to parents up front, the less defensive work you'll have to do. As educators, if we want parents to be on our team, we must initiate, define, and practice what we want that relationship to look like.

Recent research suggests that creating a partnership climate in schools can improve math proficiency for students (Sheldon, Epstein & Galindo, 2010). Schools and teachers can go beyond communication activities to engage parents as partners in supporting student math learning. Some challenges to family involvement in math learning include:

1) Math is used differently at home but teachers haven't been guided to take students' social contexts into account when planning math instruction; and

2) Most teachers have little education about how to involve parents in supporting children to extend their math skills.

Some ways that GECDSB teachers are currently building math partnerships with families include:

- Math Take Home Bags – Students take home a math bag once a week that has an activity they can engage in with their family, like measuring items around the home, reading a picture book and with prompts for math thinking, or conducting a survey of family members about a topic the class is investigating.
- Electronic Communication
- Math Goal Setting – Students take home a questionnaire to complete with their parents about the child's goals for mathematics. Parents suggest possible ways they can support their child with the goal from home. Parents are then invited in to the classroom later in the year for a demonstration by students about how they are progressing towards their goals using evidence from their work in class and at home.
- Math Nights and Workshops – Schools and teachers are hosting parents for evening sessions where they engage families together in problem solving, explain about the math program and help parents to better understand how they can help support their child's math development. Parents are also given the opportunity to provide input for the school's efforts to improve mathematics.

References

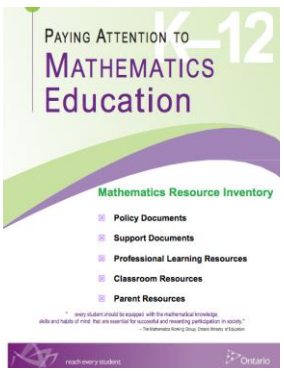
Adler, R. B., & Rodman, G. (1994). Understanding human communication. Orlando, FL: Harcourt Brace College Publishers.

More resources

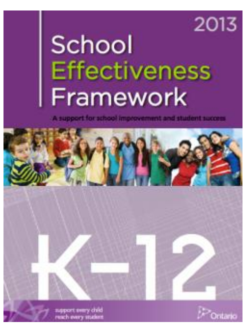
http://www.edu.gov.on.ca/eng/studentsuccess/lms/files

http://www.edu.gov.on.ca/eng/literacynumeracy/parentguidenum2012.pdf

Resources



The Ontario Ministry of Education has developed a comprehensive list of Ministry resources, research, guides and supports for educators in Ontario. Click here to access this great resource.



The School Effectiveness Framework is a tool to guide the work in our schools. Click here to see the Framework, or click here to see which areas may be particularly pertinent to mathematics.

Appendix B: Gathering Voice Data Tools

***Grades 4-12 Student* Math Survey**

	SA	S	AS	D	SD
1. I like math.					
2. I like learning math.					
3. I am good at math.					
4. Learning math makes me feel uncomfortable or nervous.					
5. Some of the math I learn in school is more important than others.					
6. In math, it is only important to get a right answer.					
7. Math is about thinking through a problem.					
8. Math is about understanding the big idea/concept.					
9. Math is about using formulas and following steps.					
10. Math is about cleverly solving problems.					
11. Math is useful and meaningful.					
12. My parents and my teachers are constantly talking about my math learning.					
13. My parents think math is important.					
14. My parents are usually able to help me with my math work.					
15. I use online homework help.					
16. I use our classroom online resources.					
17. I use other online math supports.					
18. In my class, we learn math by solving problems.					
19. After we work on a math problem in class, we usually talk about the different ways we solved the problem.					
20. I am encouraged to solve math problems many different ways.					
21. I get regular feedback about my math learning and next steps.					
22. My teacher expects me to explain how I solved a problem.					
23. In my class, all students are successful in math.					
24. I am always thinking about whether my answer makes sense or not.					
25. My teacher challenges me to do my best work in math.					
26. I can usually prove why something is right or wrong in math.					
27. I usually work with lots of other students to learn math.					
28. I think about my own math thinking.					
29. I get to talk about my math learning in my classroom.					
30. I usually use technology for math learning.					
31. I usually use math manipulatives to help me learn.					
32. I am successful in math.					

SA – Strongly Agree ***A – Agree*** ***AS – Agree Somewhat*** ***D – Disagree*** ***SD – Strongly Disagree***

***Parent/Guardian* Math Survey**

	SA	S	AS	D	SD
1. I like math.					
2. I like supporting my child in doing mathematics at home.					
3. I am good at math.					
4. Doing mathematics with my child makes me feel uncomfortable or nervous.					
5. Some kinds of math taught in school are more important than others.					
6. In math, it is only important to get a right answer.					
7. Math is about reasoning through a problem.					
8. Math is about understanding the big idea/concept.					
9. Math is about knowing formulas and procedures.					
10. Math is about strategically solving problems.					
11. Math is useful and worthwhile.					
12. I feel that I am constantly informed by my child(ren)'s school/teachers.					
13. I am satisfied with a level of communication from my child(ren)'s teachers.					
14. I feel I receive consistent communication from my child(ren)'s school/teacher about math.					
15. This year, I received these types of communications from the school.	Open Response				
16. I feel that partnership with my child(ren)'s school is important.					
17. I feel like I am fully involved in my child(ren)'s learning.					
18. I feel I have opportunities to support my child(ren)'s learning in the classroom.					
19. I feel I understand math concepts that my child is learning.					
20. I feel well-prepared to help my child with math at home.					
21. I feel I understand my child(ren)'s math homework.					
22. I set high expectations for my child(ren)'s educational achievement.					
23. I actively encourage a positive attitude towards education.					
24. I closely monitor my child(ren)'s progress at school.					
25. I contact my child(ren)'s teacher for math support.					
26. I make use of online GECDSB homework help.					
27. I use online math supports provided by the school/teacher.					
28. My child(ren) and I use other online math supports.	List				

SA – Strongly Agree A – Agree AS – Agree Somewhat D – Disagree SD – Strongly Disagree

Educator Math Survey

	SA	A	AS	D	DS
1. I like math.					
2. I like teaching math.					
3. I am good at math.					
4. Teaching math makes me feel uncomfortable or nervous.					
5. Some kinds of math taught in school are more important than others.					
6. In math, it is only important to get a right answer.					
7. Math is about reasoning through a problem.					
8. Math is about understanding the big idea/concept.					
9. Math includes using formulas and procedures.					
10. Math is about strategically solving problems.					
11. Math is useful and worthwhile.					
12. I consistently communicate with parents.					
13. I consistently engage with parents about math.					
14. List the various methods that you use to engage **parents** in their child(ren)'s math learning.	(Open Response)				
15. I feel that partnership with parents is important.					
16. I feel like I engage parents as partners in learning.					
17. I create opportunities for parents to support their child(ren)'s learning in the classroom.					
18. I feel my students are prepared to learn math.					
19. I regularly consolidate math learning.					
20. I consistently use assessment *for* learning in math.					
21. I constantly use assessment *of* learning in math.					
22. I consistently use flexible groupings of students.					
23. I regularly prompt for metacognition in math.					
24. I consistently use a three part math lesson.					
25. I frequently use a variety of math resources.					
26. I constantly facilitate math talk in the classroom.					
27. I usually use technology for math learning.					
28. I consistently use manipulatives in my lessons.					
29. My math instruction results in success for my students with learning disabilities.					
30. My lessons focus on building understanding of the math concepts.					
31. I feel that my students are successful in learning math.					
32. I have strong understanding of math concepts that I teach.					
33. I have a strong understanding of math procedures.					
34. I have a strong productive disposition towards math teaching.					
35. I have strong understanding of ways to formulate, represent and solve math problems.					
36. I have strong capacity for logical thought, reflection and justification in math.					
37. I feel supported in my professional learning of math.					
38. I have the resources I need to support math teaching/learning.					
39. I am presently involved in math professional development.					
40. My preferred learning structure(s) are:	(Open Response)				
41. I would like professional development in math content.					
42. I would like professional development about math pedagogy.					
43. Comments	(Open Response)				

SA – Strongly Agree   A – Agree   AS – Agree Somewhat   D – Disagree   SD – Strongly Disagree

***Administrator* Math Survey**

	SA	A	AS	D	DS
1. I like math.					
2. I like teaching math.					
3. I am good at math.					
4. Math makes me feel uncomfortable or nervous.					
5. Some kinds of math taught in school are more important than others.					
6. In math, it is only important to get a right answer.					
7. Math is about reasoning through a problem.					
8. Math is about understanding the big idea/concept.					
9. Math includes using formulas and procedures.					
10. Math is about strategically solving problems.					
11. Math is useful and worthwhile.					
12. I consistently communicate with parents.					
13. I observe teachers consistently communicating with parents.					
14. I consistently communicate with parents about math.	(Open Response)				
15. I observe teachers consistently communicating about math with parents.					
16. List the various methods used in your school to engage **parents** in their child(ren)'s math learning.					
17. I feel that partnership with parents is important.					
18. I feel I engage parents as partners in learning.					
19. I create opportunities for parents to support their child(ren)'s learning in the classroom.					
20. I constantly notice students engaged in problem solving.					
21. I consistently notice students engaged in consolidation math learning.					
22. I notice consistent assessment *for* learning in math.					
23. I notice consistent use of assessment *of* learning in math.					
24. I notice students consistently working in flexible groupings.					
25. I notice most/all students engaged in metacognition in math.					
26. I notice most/all students engaged in the three part math lesson.					
27. I notice the consistent use of a variety of resources.					
28. I notice consistent use of technology in math learning.					
29. I consistently see students using manipulatives.					
30. I notice most/all students engaged in math talk in the classroom.					
31. I notice math success for most/all students with learning disabilities.					
32. I have strong understanding of math concepts.					
33. I have a strong understanding of math procedures.					
34. I have a strong productive disposition toward math.					
35. I have strong understanding of ways to formulate, represent and solve problems.					
36. I have strong capacity for logical thought, reflection and justification in math.					
37. I have the resources I need to support math teaching/learning.					
38. I feel ready to lead math teaching and learning in my school.					
39. I feel prepared to effectively support the GECDSB math vision.					
40. I feel supported in leading math learning.	(Open Response)				
41. I am presently involved in math professional development.					
42. My preferred learning structure(s) are: (Open response)					
43. I am presently supporting math professional development in my school.	(Open Response)				
44. I would like professional development in math content.					

Student Interview Questions

1. *These questions are about mathematical mindsets, attitudes and beliefs of stakeholders.*
    a. In your opinion, what does it mean to do math?
    b. In your opinion, what does it mean to be good at math?
    c. Do you feel you are good at math?
    If yes, why do you feel that way?
    If no, can you identify the barriers?

2. *These questions are about being a partner in learning.* ***A partnership is where two or more people or groups work collaboratively toward mutual goals.***
    a. Do you see yourself as a partner in education? Y/N
    b. Who are your partners?
    c. Describe your partnerships.

3. *These questions are about communication between home and school that you experienced.*
    a. Based on your experience, how is math learning supported between home and school?
    b. What specific does your family and teachers use to communicate between home-school?
    c. What could be done to better support your learning between home-school?

4. *This question is about voice which means choices or having a say in your learning*.
    a. Do you feel you have a voice in your learning?

5. If you could send a message to other kids, parents, teachers or other people in charge of schools about math learning, what would you like them to know?

Parent Interview Questions

1. *These questions are about mathematical mindsets, attitudes and beliefs of stakeholders.*
    d. In your opinion, what does it mean to do math?
    e. What do you think your (students/child(ren)) would say it means to be good at math?
    f. In your opinion, what does it mean to be good at math?
    g. Do you feel you are good at math?
    If yes, why do you feel that way?
    If no, can you identify the barriers?

2. *These questions are about being a partner in learning.* ***A partnership is where two or more people or groups work collaboratively toward mutual goals.***
    d. Do you see yourself as a partner in education? Y/N
    e. Who are your partners?
    f. Describe your partnerships.
    g. What would your ideal partnership include?

3. *These questions are about communication between home and school that you experienced.*
    a. Based on your experience, how is math learning supported between home and school?
    b. What specific methods of communication are used between home-school?
    c. What could be done to better support a home-school connection?

4. Any other anecdotes

Educator Interview Questions

1. *These questions are about mathematical mindsets, attitudes and beliefs of stakeholders.*
    a. In your opinion, what does it mean to do math?
    b. What do you think your (students/child(ren)) would say it means to be good at math?
    c. In your opinion, what does it mean to be good at math?
    d. Do you feel you are good at math?
    If yes, why do you feel that way?
    If no, can you identify the barriers?

2. *These questions are about being a partner in learning.* ***A partnership is where two or more people or groups work collaboratively toward mutual goals.***
    a. Do you see yourself as a partner in education? Y/N
    b. Who are your partners?
    c. Describe your partnerships.
    d. What would your ideal partnership include?

3. *These questions are about communication between home and school that you experienced.*
    a. Based on your experience, how is math learning supported between home and school?
    b. What specific methods of communication are used between home-school?
    c. What could be done to better support a home-school connection?

4. *These questions are about your professional learning experiences and preferences and system supports.*
    a. What supports do you think teachers need to improve teaching and learning in mathematics?
    b. If you can, describe an experience of a meaningful professional learning and how this learning has changed your practice.
5. Any other anecdotes you would like to share.

Administrator Interview Questions

1. *These questions are about mathematical mindsets, attitudes and beliefs of stakeholders.*
    a. In your opinion, what does it mean to do math?
    b. What do you think your (students/child(ren)) would say it means to be good at math?
    c. In your opinion, what does it mean to be good at math?
    d. Do you feel you are good at math?
    If yes, why do you feel that way?
    If no, can you identify the barriers?

2. *These questions are about being a partner in learning.* ***A partnership is where two or more people or groups work collaboratively toward mutual goals.***
    a. Do you see yourself as a partner in education? Y/N
    b. Who are your partners?
    c. Describe your partnerships.
    d. What would your ideal partnership include?

3. *These questions are about communication between home and school that you experienced.*
    a. Based on your experience, how is math learning supported between home and school?
    b. What specific methods of communication are used between home-school?
    c. What could be done to better support a home-school connection?

4. *These questions are about your professional learning experiences and preferences and system supports.*
    a. Recently, mathematics came into the focus of the Ministry vision for the 21st century education. Do you feel as though you can fully support this vision, meaning mathematics teaching/learning in your school?
    b. What supports do you think teachers need to improve teaching and learning in mathematics?
    c. If you can, describe an experience of a meaningful professional learning and how this learning has changed your practice.
5. Any other anecdotes you would like to share.

APPENDIX C: Learning Environment Data Protocols and Tools

EMAIL TO PRINCIPALS

Greetings!

The Learning Environment committee, a part of the Math Task Force, is interested in visiting classrooms in our system to better understand what encompasses a *Responsive Math Learning Environment*.

We are contacting you because your school is amongst a group of schools selected to be part of the learning for the Math Task Force this year. We plan to gather data and evidence during classroom visits from some of these schools for deeper observation which will lead to board wide learning.

In the next few days, the teachers at your school will receive a personal email to invite them to voluntarily host a visit from our MTF Learning Environment Committee team. The email will include an outline of the process of the visits and a copy of a student survey (see attached).

Teachers who express their interest will receive a student survey link for their class to complete. The individual class results will be shared with all teachers.

By the beginning of February, our team will notify the teachers and principals of the schools that we will be visiting.

Should you have any questions, you are welcome to contact my office or Fouada Hamzeh or Brenda DelDuca in the Program Department.

Sincerely,

Clara

Clara Howitt
Superintendent of Education

EMAIL TO TEACHERS

Greetings Teachers!

This year our Board has established a Math Task Force for the purpose of digging deeper into what factors influence mathematics learning. Our task force is divided into five sub-committees: Professional Learning, System Practices, Reciprocal Partnerships, Curriculum and Resources, and Responsive Learning Environment.

The Responsive Learning Environment committee is interested in visiting classrooms in our system to better understand what encompasses a *Responsive Math Learning Environment*. We are contacting you because your school is amongst a group of schools selected to be part of the learning for the Math Task Force this year. During classroom visits we will gain a deeper understanding through observations of the mathematics learning environment, which will lead to board-wide learning. The underlying assumption for the visit is that the team, teachers and students will work together to create some new knowledge — we are in it together. The observation is a shared experience, and so is the debriefing.

We are emailing you directly to invite you to consider volunteering to host a visit from our Learning Environment committee (3-4 members). The Classroom Observation Visit begins with a pre-meeting with you so we can share the details of the visit. The visit is comprised of 30-40 minutes of observation followed by a debrief. The visits are intended to be an open conversation between the team, teachers and students. Half day release will be provided for the debrief session. Please find attached the outline for the visit for your consideration.

Please fill out the brief survey by January 17th if you wish to express your interest in hosting our team. Once we receive your declaration of interest we will email you a link to the student survey for your class to complete. Please find a draft of the student survey attached for your records. The individual class results will be shared with all teachers.

By the beginning of February, our team will notify the teachers and principals of the schools that we will be visiting.

Should you have any questions, you are welcome to contact my office or Fouada Hamzeh or Brenda Del Duca in the Program Department.

Classroom Visits- A Collaborative Learning Conversation around Mathematics Learning Environments

Observation Protocol

This Protocol was adapted from the "Interesting Moments" Peer Observation Protocols created by the National School Reform Faculty and Data-in-a-Day by Margery Ginsberg.

The underlying assumption for this protocol is that the observer and the observed will work together to create some new knowledge — they are in it together. The observation is a shared experience, and so is the debriefing.

Pre-Observation Conference

A pre-observation meeting with the host teacher is planned to build trust and comfort between the visitors and the host teacher.

Observation

The visitor maintains an open field of vision, recording specific examples that illustrate the learning environment. (e.g. descriptive observations, diagrams, direct quotes) — anything that may lead to "deep" wonderings.

Debriefing

Visitors, host teachers and students reflect as a group on the noticings and wonderings that were documented. A conversation develops between all stakeholders as themes unfold.

Note: Prerequisite for this protocol is a high level of trust between the two participants: trust that the debriefing is not about evaluation; trust that each will be thoughtful, will listen and respond to the other; trust that whatever knowledge is created will be shared knowledge.

CLASSROOM VISIT PROTOCOL

Pre-Observation Conference

- Two committee members will visit teachers for a pre-meeting to build trust and comfort and share the protocol
- Student surveys will be shared electronically with the teacher in order for students to complete them before the visit. Survey data will be shared with the teacher before the visit.

Observations

- Teachers are consulted about choosing students that will be comfortable in engaging in a conversation during the classroom visit. The teacher will invite up to four students to be part of the debriefing process.
- The visitors will enter the room with the three realms fresh in their minds (see attachment).
- Observations will take 30- 40 minutes.
- Visitors will refrain from talking to the teacher during the observations until the debrief.
- All conversations with students will happen in a 1:1 ratio. All questions directed to students will be based on immediate classroom observations.
- The team will divide into pairs to observe together. One person will record observations about the physical environment while the other records student observations. Partners will vet each other's observations before recording another observation.

Debrief

- The observation team will review their data to ensure an asset-based debrief.
- The team will share their data with each other to determine ideas they would like to learn more about which were uncovered from the documented "noticings" and "wonderings".
- Each partner team will highlight asset-based observations that connect to a responsive learning environment and transpose these onto post-its. The post-its will be clustered together to uncover themes. Wonderings will be derived from these themes.
- The teacher, administrator and up to 4 students will join the debrief.
- The team of visitors, teacher, administrator and students will engage in a collaborative learning conversation around their math learning environment.
- One member of the visiting team will act as recorder of the answers and the questions (if students and teachers give consent, a video will be recorded).
- More or fewer questions may arise or be asked as the conversation unfolds.
- One committee member will act as the moderator during the conversation to keep a focus on the data rooted in the observation and to allow for equity of voice.

- During the debrief, the subsequent questions will be recorded and during the break the visitors will review the questions before the interview commences.
- Time will be allowed to discuss survey results.

The committee will invite the students to complete the focus questions:

Physical: "If you were to design the optimal physical space to help you learn math, what would it look like?"
Social/Emotional: "What emotions do you feel when you are learning math? Describe why you feel this way."
Choice/Voice: "Is it important to have choice during your math learning and to have your voice heard?"
Overall Question: "What helps you learn math?"

Team and Educator Reflection

Reflective Prompts about the Protocol

a. How did the protocol work for you?
b. What would need to change to make this process more meaningful for our intentions?
c. How was learning honoured through this process?

Reflective Prompts about the Learning Environment (3 Realms)

a. What is one thing that most surprised you in our work today?
b. How has your understanding about the learning environment changed after today's learning?
c. What is one thing you will do differently because of today's learning?

Before we leave the school, our team will critically revisit the research to reflect upon how the visit informed our findings, supported the research, or gave rise to other additions that we need to consider.

LEARNING ENVIRONMENT STUDENT SURVEY

INSTRUCTIONS:

The survey contains statements about practices that could take place in your class. There are no '"right or "wrong" answers; your opinion is what matters. Think about how well each statement describes what this class is like for you.

This survey will be used by your teacher's learning team to help understand how the classroom environment impacts your math learning. The responses to these questions are anonymous and the classroom results will be available for you, your class, and your teacher to see.

	Almost Never	**Seldom**	**Sometimes**	**Often**	**Almost Always**
1. In this class, I know that I can find help when I need it.	1	2	3	4	5
2. In this class, I feel comfortable to take risks and share my math thinking.	1	2	3	4	5
3. In this class, I learn how math helps me in everyday life	1	2	3	4	5
4. In this class, there are math materials available for us to use:					
A) Manipulatives	1	2	3	4	5
B) Posted Charts	1	2	3	4	5
C) Calculator	1	2	3	4	5
D) Math Talk	1	2	3	4	5
E) Electronics (iPad, computers)	1	2	3	4	5
F) Text Book	1	2	3	4	5
G) Internet to explore information related to mathematics	1	2	3	4	5
5. In this class, it is acceptable to express my own opinion.	1	2	3	4	5
6. In this class, I have the chance to choose who to work with when solving math tasks.	1	2	3	4	5
7. In this class, I have the chance to choose any method to solve math tasks.	1	2	3	4	5
8. In this class, we all support each other with our math learning.	1	2	3	4	5
9. In this class, I see that my math thinking (e.g., words, work, voice) is visible.	1	2	3	4	5
10. In this class, I feel my ideas are valued as much as everybody else's are.	1	2	3	4	5
11. In this class, I know how to use materials to explore math.	1	2	3	4	5
12. The layout of my class makes learning math comfortable.	1	2	3	4	5

APPENDIX D: Learning Briefs

GREATER ESSEX COUNTY DISTRICT SCHOOL BOARD

Creating International Conversations

The GECDSB believes in learning together and seeks out opportunities to build conversations with our international partners. The Reciprocal Learning Program is an initiative developed between the University of Windsor, the Greater Essex County District School Board, Southwest University, and Chong Qing schools. This partnership has helped to shift the conversation from: comparison to collaboration; from obstacles to opportunities; and from limits to learning.

The Greater Essex County District School Board provides mathematics education that engages and empowers students through collaboration, communication, inquiry, critical thinking, and problem-solving, to support each student's learning and nurture a positive attitude towards mathematics.

GECDSB, A Vision for Mathematics, 2016

The purpose of these learning briefs is to share the research, discussion and insight garnered from the intensive work of the Greater Essex County District School Board's Math Task Force. These papers are rooted in the GECDSB core beliefs, the Full-Day Early Learning—Kindergarten program and the Ontario Mathematics Curricula for grades 1-8, 9-10, and 11 & 12. The briefs are meant to elevate, enrich and extend the discourse of mathematics education with the intention of encouraging a positive and productive disposition toward mathematics for all learners.

Each paper provides a list of sources to extend the professional conversation and enhance the learning. In addition, live links appear at the end of the papers with connections to various resources.

CREATING INTERNATIONAL CONVERSATIONS

Mathematics achievement is a global focus. Sound bites in the media share bits of data from international reports that rank and compare countries. However, when conclusions about student achievement are limited to 140 characters, we miss the fullness, insight and depth of the evidence. A comparison between nations that examines achievement scores in isolation of the culture and context is incomplete and ultimately inadequate in informing the work of educators.

The Reciprocal Learning Program is an initiative developed between the University of Windsor, the Greater Essex County District School Board, Southwest University, and Chong Qing schools, which has become the essential part of a 7-year Canada-China Reciprocal Learning Partnership Project funded by the Social Sciences and Humanities Research Council of Canada (SSHRC) with five other Canadian and Chinese partner institutions (Xu & Connelly, 2013). The goals of the Reciprocal Learning Program are: to provide an exceptional learning experience for teacher candidates, educators and administrators; to expand perspectives regarding societies of increased diversity; to foster international collaboration among faculty members who are interested in cross-cultural studies; to promote multicultural education; and to enhance international education relationships (Xu, 2011). The core of the Reciprocal Learning Program and the SSHRC Partnership Grant project rests in the reciprocity between educators, parents and students from both nations. This partnership has helped to shift the conversation from: comparison to collaboration; from obstacles to opportunities; and from limits to learning.

From Comparison to Collaboration

In 2012, the Organization for Economic Cooperation and Development (OECD) released results from the Program for International Student Assessment (PISA) which demonstrated a slide in Canadian students' ranking. At that time, the former Deputy Prime Minister John Manley was quoted in the Globe and Mail as saying that the performance of Canadian students was "on the scale of a national emergency" (Alphanso, 2013). This statement fueled a frenzy of media attention on the Canadian "math problem" and sparked controversy over the ways in which we can emulate the high-ranking Chinese education system.

One of the main foci of the media attention was on the way in which Chinese math teaching focused on "rote learning" which was erroneously positioned in opposition to "discovery learning." This drove a fruitless public debate, distracting conversations from a thorough analysis of the data and the learning that could have been garnered.

Countries differ in such global characteristics as the centralization of educational policies, the organization and types of schools, and the success of efforts to provide universal access to education. The status of teaching in the society, the composition and mobility of the student population, and the extent to which external examinations determine one's life chances, all constrain the ways in which mathematics is taught and learned.

(National Research Council, 2001, p. 31)

A comparison between nations that examines achievement scores in isolation of the culture and context is incomplete and ultimately inadequate in informing the work of educators.

This partnership has helped to shift the conversation from: comparison to collaboration; from obstacles to opportunities; and from limits to learning.

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Assessments like PISA can generate discussion, inferences and further investigation. There is unquestionably information to be uncovered. However, attempts at comparisons between the education practices of nations are at best interesting and at worst a red herring. When we examine practices in isolation of their context, we tell an incomplete and incoherent story. These comparisons fall drastically short of identifying which practices are responsible for student achievement. Therefore, we must dig deeper into the evidence in order to help guide our educational improvements.

The values of a nation inform their systems of education. What is taught, who is taught and who does the teaching are simple examples of how the ideals and structures of mathematics education are tied to the complex cultural conditions of each nation. Certainly, the learning experience in Canada is not the same as the learning experience in China; but the living experience in Canada is not the same as the living experience in China. Each country provides a unique context for mathematics learning and when we compare stories, our learning needs to be positioned in an understanding of this context.

The results from PISA and other international data identifies the Canadian (Ontario) education model as world leading with respect to critical criteria such as equity, inclusiveness and support for all learners – criteria which define our national values. Our educational system is also lauded for the richness, diversity and comprehensiveness of the curricula (Council of Ministers of Education of Canada, 2012).

Insights from the Project

"It is not about copying and borrowing; it is about relating and understanding" was a statement made by Dr. ShiJing Xu, Associate Professor, University of Windsor, during a presentation about the learning of the Reciprocal Partnerships Project (2016). One of the great understandings that has come from the project is this idea that mathematics education is rooted in a culture, and no one practice can be isolated and replicated with the expectation of similar results in student achievement.

Through conversation, educators are able to build a better understanding of practices that support student learning. These cross-cultural perspectives support new approaches to research on curriculum and their application in classrooms. The insights from the project participants are demonstrated through their reflections. Educators' feedback focused on to the overall social standard of mathematics. They noticed the prominent social standard of mathematics among their Chinese partners, and observed how this focus influences the systems within the schools. Our values identify our priorities. Thus, an intentional and thoughtful prioritization of mathematics education will certainly serve to direct our energy to its service.

In order to capitalize on the potential for improving mathematics, we need to understand the entire story. Although the data may start the discussion, the learning is in the details. Through our international partnerships, we gain insight and understanding. Once we appreciate the "why" and "how," we can move the conversation toward "let's try." *The learning* is the heart of this partnership, because when we learn together, we can learn far more than when we are learning alone.

The values of a nation inform their systems of education. What is taught, who is taught and who does the teaching are simple examples of how the ideals and structures of mathematics education are tied to the complex cultural conditions of each nation.

"It is not about copying and borrowing; it is about relating and understanding" by Dr. ShiJing Xu

The learning is the heart of this partnership, because when we learn together, we can learn far more than when we are learning alone.

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GREATER ESSEX COUNTY District School Board

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Globe and Mail - What Shanghai Can Teach Us about Teaching Math
http://www.theglobeandmail.com/news/national/education/what-shanghai-can-teach-us-about-teaching-math/article17835021/?page=all

OECD - Programme for International Student Assessment (PISA)
https://www.oecd.org/pisa/

Doing Math with Your Child
http://www.edugains.ca/newsite/math/offeredotherlanguage.html

GREATER ESSEX COUNTY
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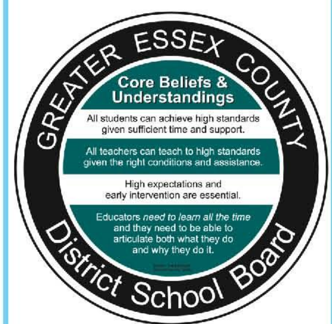
Expertise of All

The GECDSB values mathematical expertise and believes in building content and pedagogical knowledge of all educators. A mathematics teaching model which values mathematics expertise for all teachers, supports the tenets of the GECDSB core beliefs. We believe it is the learner who will become the expert, and at the GECDSB we are *all* learners.



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GECDSB, A Vision for Mathematics, 2016

The purpose of these learning briefs is to share the research, discussion and insight garnered from the intensive work of the Greater Essex County District School Board's Math Task Force. These papers are rooted in the GECDSB core beliefs, These papers are rooted in the GECDSB core beliefs, the Full-Day Early Learning—Kindergarten program and the Ontario Mathematics Curricula for grades 1-8, 9-10, and 11 & 12. The briefs are meant to elevate, enrich and extend the discourse of mathematics education with the intention of encouraging a positive and productive disposition toward mathematics for all learners.

Each paper provides a list of sources to extend the professional conversation and enhance the learning. In addition, live links appear at the end of the papers with connections to various resources.

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Teaching expertise extends beyond content area knowledge. Pedagogical content knowledge is a highly specialized skill-set that differs from subject specific knowledge and general pedagogical knowledge (Shulman, 1987).

EXPERTISE OF ALL

The National Council of Teachers of Mathematics (NCTM, 2000) has cited that teacher knowledge, attitide and skill are of central importance in the teaching of mathematics. It is therefore no surprise that the role of content expertise is a critical debate among educators, parents and policy makers. In order for our students to become successful citizens of the 21st century, it is vital that we educate children to become proficient mathematicians and our success is in large part determined by effective teaching. The concept of *mathematics teaching expertise* requires careful study, and a review of the research raises many significant considerations that need to be closely examined at a classroom, school and system level.

Experience, Experts and Expertise

In order to better understand the crux of the issue, we need to explore the concepts of experience, experts and expertise. According to John Hattie, Professor of Education and Director of the Melbourne Education Research Institute in Melbourne, Australia, experienced teachers are those who have years of practice and familiarity in teaching, however Hattie identifies that experience and experts are not necessarily one in the same. He states:

> *Experts and experienced teachers do not differ in the amount of knowledge they have about curriculum matters or knowledge about teaching strategies. But experts do differ in how they organize and use this content knowledge.* (Hattie, 2003)

Certainly within the content rich area of mathematics there is a desire to identify and leverage our expert teachers. Conversations continue about math experts and their roles in schools. In order to best serve the interests of our students we need to expand our definition of *expert* to include *expertise*. If an expert is only defined as someone who has formal education in a content area, then we are missing crucial aspects of teaching expertise.

The well-known work of Lee Shulman (1987) on pedagogical-content-knowledge (PCK) has long demonstrated that teaching expertise extends beyond content area knowledge. Shulman describes content as the "what" and pedagogy as the "how" of teaching. According to his work, pedagogical content knowledge is a highly specialized skill-set that differs from subject specific knowledge and general pedagogical knowledge (Shulman, 1987). Within the context of mathematics teaching we need to adopt a comprehensive definition of the term "expert" which is grounded in research and inclusive of all the domains of expertise.

The use of specialty mathematics teachers is an accepted practice in secondary schools and in some elementary schools who adopt a "rotary" model. This model can have advantages when specialty teachers have demonstrated expertise, but the limitations of this model must be critically considered. The drawbacks include logistical implications for staffing and scheduling as well as the isolation of mathematics instruction to specific blocks of time. This structure impairs the opportunity for meaningful integration of mathematics throughout the instructional day, and this integration is a central principle of the Ontario Mathematics Curriculum. In addition, this model restricts professional learning to only specialty teachers, creating significant long-term impacts on schools and a school-system (Gerretson, Bosnick & Schofield, 2008).

Students engage in mathematics learning throughout their years of elementary and secondary school. Effectively supporting this learning requires all teachers to continue to develop and refine their expertise in mathematics. It is critical for our schools and school system to support deeper and broader understanding of mathematics teaching and learning for *all* educators.

Democratization of Mathematics

The democratic values of education are echoed in the core beliefs of the Greater Essex County District School Board, which

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state that "all students can achieve high standards given sufficient time and support," and that "all teachers can teach to high standards given the right conditions and assistance." A mathematics teaching model, which only values mathematics expertise for specific teachers, defies the tenets of the GECDSB core beliefs. In addition, a system that reserves math expertise for only a few perpetuates a culture in which it is acceptable to claim mathematical illiteracy.

Currently, we experience a cultural norm in which it is acceptable to state "I can't do math," however the same posturing is not accepted in literacy. It is uncommon to hear a person publicly announce one's illiteracy, saying "I can't read." In order to challenge these cultural norms and advance the current social standards of mathematics proficiency, we need to remain committed to our core beliefs and demonstrate through our actions as educators that we believe all children can learn math and all teachers can teach math.

Expertise as Proficiency

Expertise should be defined through the lens of math proficiency (National Research Council, 2001). Just as literacy is more than the mere decoding of words, proficiency in mathematics is more than the mere recall of tables, facts, and formulas. Mathematics proficiency as defined in the GECDSB Vision for Mathematics includes five interwoven threads of mathematics skill: adaptive reasoning, strategic competence, procedural fluency, conceptual understanding, and productive disposition. This definition of mathematics proficiency is research–based, comprehensive and reflects our most current understanding of what students need as 21st century learners *and* leaders. Yet, our current reality stands before us.

Mathematics teaching should not look the same as it did decades ago. Years of research has informed our current best practices. Teachers in today's classrooms need to teach math in a way that may be different from how they were once taught. Previous definitions of what it meant to "do and be good at math" have resulted in some people developing limited fluency of mathematical procedures and fragmented understanding of mathematical concepts. In addition, mathematics learning which was exclusively based on rule-learning, speed and accuracy, and devoid of conceptual connections, has given rise to math anxieties and fixed math mindsets (Boaler, 2015). The focus of GECDSB over the past several years has been on developing growth mindsets. Our current challenge is to extend this work to the other mathematical proficiencies in order to develop the expertise of all.

Building Excellence by Building Expertise

Each and every day, our teachers rise to meet the challenge of their calling. We are learners and we believe that "educators need to learn all the time and they need to be able to articulate both what they do and why they do it." Teachers require time, support and resources to continue, extend and sustain the great work which is happening across our system. We must value the expertise of our greatest resource – our educators.

We trust that through our collective and collaborative learning we will find the answers we seek. Expertise is built through learning, and learning is what sustains, improves and empowers us. The GECDSB believes in building the capacity of all educators. We believe that it is *the learner* who will become the expert…and *we are all learners*.

Effectively supporting the learning requires all teachers to continue to develop and refine expertise in mathematics.

In order to challenge these cultural norms and advance the current social standards of mathematics proficiency, we need to remain committed to our core beliefs and demonstrate through our actions as educators that we believe all children can learn math and all teachers can teach math.

Expertise should be defined through the lens of math proficiency: adaptive reasoning, strategic competence, procedural fluency, conceptual understanding, and productive disposition.

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LINKS

Seven Foundational Principles for Improvement in Mathematics, K–12
https://www.edu.gov.on.ca/eng/teachers/studentsuccess/FoundationPrincipals.pdf

National Council of Teachers of Mathematics
http://www.nctm.org/

Lucy West - Building Success in Mathematics
https://vimeo.com/153792153

Marian Small—The Art of Mathematics
https://vimeo.com/103956482

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https://www.youtube.com/watch?v=gGgH3RMqg9U

GREATER ESSEX COUNTY DISTRICT SCHOOL BOARD

Rote versus Discovery: Moving the Discussion Forward

The conversation of mathematics proficiency cannot be framed in the context of rote versus discovery learning because neither defines nor describes mathematical proficiency. The GECDSB believes in an integrated approach to teaching and learning that is responsive to the individual needs of the learners and is rooted in a conversation about mathematical proficiency.

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GECDSB, A Vision for Mathematics, 2016

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Interestingly, what divides them is a common understanding of what it means to "do and be good at math." It is this shared definition of mathematical proficiency that ultimately bridges the divide.

The conversation of mathematics proficiency cannot be framed in the context of rote versus discovery learning because neither defines nor describes proficiency.

ROTE VERSUS DISCOVERY: MOVING THE CONVERSATION FORWARD

The rote versus discovery debate has occupied public discussion for years. On either side of the argument stands a passionate group with extensive research that claims to support their position. Each group believes that mathematics is important. Each group wants what is best for students. Interestingly, what divides them is a common understanding of what it means to "do and be good at math." It is this shared definition of mathematical proficiency that ultimately bridges the divide.

Daniel Ansari, of the University of Western Ontario, is professor of psychology and Canadian Research Chair of Developmental Cognitive Neuroscience. His work uses behavioural research methods and neuro-imaging to build an understanding of how children learn about numbers. Ansari (2015) recently published a compelling article with the Canadian Education Association which called for a truce to the "math wars". Ansari (2015) drew attention to the "false dichotomy" that is the math wars, stating that "these two approaches are frequently painted as being completely distinct and diametrically opposed to one another, creating the perception that there is a need to side with one particular view of best practice in math education".

Within this conversation is an array of terminology; words and phrases that unless clearly defined, lead us to talk in circles. Our dialogue must begin by operationalizing the terms being used, or the discourse becomes futile. Ansari describes *rote learning* as being synonymous with the rehearsing or drilling arithmetic facts and *discovery learning* as incorporating the underlying principles of mathematics through hands-on activities and open-problem solving. In the scope of mathematics education we can see how both of these narrow views fall desperately short of defining mathematics proficiency. Neither gives us a start or end point. The conversation of mathematics proficiency cannot be framed in the context of rote versus discovery learning because neither defines nor describes proficiency. Through the article, Ansari repositions the conversation as one of procedural and conceptual knowledge and argues that both are important parts of mathematics. He also calls for education stakeholders to abandon these emotionally-charged debates and use evidence to inform their dialogue.

The Greater Essex County District School Board includes procedural fluency and conceptual understanding as part of its vision for mathematics, but extends the definition based on the broad research of the National Research Council publication, *Adding It Up* (2001). Proficiency in mathematics is defined as: procedural fluency, conceptual understanding, strategic competence, adaptive reasoning, and productive disposition. Based on this definition, our work becomes designing instruction that mobilizes a range of strategies in order to move students toward proficiency (National Research Council, 2001).

Mathematics Proficiency

I want my child to know her times tables. *Absolutely*. I want my son to understand the concept of number. *Of course*. I want my students to solve problems using multiple strategies. *Definitely*. I want my daughter to love and excel in math. *Certainly*. I want my students to think mathematically and be able to justify their thinking. *Yes, without a doubt!*

Proficiency in Mathematics cannot be defined by any one facet, application, strategy, or attitude. It is an interweaving of five competencies, each distinct but with no one strand encompassing the entirety (National Research Council, 2001). It is the entwining of the threads that becomes the framework for mathematics proficiency and this is grounded in the goals and expectations of the Ontario Curriculum grades 1-8: Mathematics (2005). The proficiencies have been described in great detail in the publication *Adding It Up*, where the authors boldly state:

> *The most important observation we make here, one stressed throughout this report, is that the five stands are interwoven and interdependent in the development of proficiency. Mathematical proficiency is not a one dimensional trait, and it cannot be achieved by focusing on just one or two of the strands. (National Research Council, 2001).*

The Five Threads of Proficiency

Skemp (1976) argued that it is not enough for students to understand *how* to perform various mathematical tasks; they must understand *why*. He used the term "relational understanding" and explained that it is an appreciation of the underpinnings, ideas and relationships in mathematics. The first of the threads of proficiency is conceptual understanding, which is the *why* of math. It is the ability to understand mathematical concepts, operations, and relationships, and the contexts in which they are useful. For example, when considering a multiplication question such as 55x24, a person with conceptual understanding can see that the problem could be represented as repeated addition, or as the area of a quadrilateral, the number of seats in a theatre, and any other scenario they can conceive.

Students with conceptual understanding are able to arrange representations in a variety of ways and use these representations to build new ideas. They can discuss the similarities or differences among these representations and make connections between "clusters" of mathematical principles, laws and properties (National Research Council, 2001, p. 120).

Building on this idea is the second thread of procedural fluency. This is the skill of carrying out procedures flexibly, accurately, and efficiently, and understanding the context in which the procedures should be applied. In the example of 55x24, a person with procedural fluency may apply a known method such as organizing the numbers horizontally and carrying out a standard algorithm. Being able to estimate and complete mental computations is also an important part of procedural fluency. Students need to be efficient and accurate in performing basic computations and a good conceptual understanding helps to support procedural fluency.

In school mathematics, procedural fluency and conceptual understanding are sometimes positioned as opposing concepts. This could not be further from the truth. The authors of *Adding it Up* clarify:

> *Procedural fluency and conceptual understanding are often seen as competing for attention in school mathematics. But pitting skill against understanding creates a false dichotomy. As noted earlier, these two are interwoven. Understanding makes learning skills easier, less susceptible to common errors and less prone to forgetting. By the same token, a certain level of skill is required to learn many mathematical concepts with understanding, and using procedures can help strengthen and develop that understanding* (National Research Council, 2001, p. 122).

Being able to solve mathematical problems is a large part of what it means to be proficient in mathematics. The third thread of proficiency is strategic competence, which is the ability to formulate, represent and solve mathematical problems using effective strategies. Devising a strategy includes being able to manipulate the process of problem-solving by formulating and selecting approaches. Students with strategic competence will exhibit conceptual understanding when they select and organize their solution, and procedural fluency when they carry out their strategy with efficiency. Strategic competence is an integral part of procedural fluency because over time and with experience, students see the value of selectiveness and efficiency of procedures. For example, consider when it is useful to multiply instead of adding repeatedly. Students need to be able to "replace by more concise and efficient procedures, those cumbersome procedures that might at first have been helpful in understanding the operation" (National Research Council, 2001. p. 126).

The fourth thread of proficiency, adaptive reasoning, is the capacity for logical thought, reflection, explanation, and justification. It is not enough to just select and carry out a strategy. Deductive reasoning is used to make conclusions using facts, definitions, rules, or properties. Mathematics learning develops when people are able to articulate the proofs and mathematical decisions they made, including: why a certain strategy was selected, why it was the most effective, and how they know they were successful or not. With the assistance of representations, even young children can demonstrate their justifications and reasoning. It is important to consider that, "it is not sufficient to justify a procedure just once… Students need to use new concepts and procedures for some time and to explain and justify them by relating them to concepts and procedures they already understand" (National Research Council, 2001, p. 130).

There has been significant work done in the area of Mathematical Mindsets by leaders like Jo Boaler (2015), who explain how our beliefs are strongly tied to our behaviour. Thus, seeing mathematics as useful and worthwhile helps to empower children to engage deeply in their learning. The fifth thread, productive disposition, is an inclination to see mathematics as beneficial and valuable. It allows students to see where and how mathematics can be applied, not only to the world around them, but in service of the intrinsic beauty of the discipline. Productive disposition is a tenacious belief that mathematics is not arbitrary or irrelevant, but understandable and worth the effort.
Developing a productive disposition does not

Conceptual understanding is the ability to understand mathematical concepts, operations, and relationships, and the contexts in which they are useful.

Procedural fluency is the skill of carrying out procedures flexibly, accurately, and efficiently, and understanding the context in which the procedures should be applied.

Strategic competence is the ability to formulate, represent and solve mathematical problems using effective strategies.

Adaptive reasoning is the capacity for logical thought, reflection, explanation, and justification.

Productive disposition is an inclination to see mathematics as beneficial and valuable. It allows students to see where and how mathematics can be applied, not only to the world around them, but in service of the intrinsic beauty of the discipline.

Proficiency develops over time, with practice, instruction, feedback, support and opportunity. As educators we take up the challenge of synchronically developing each of the strands of proficiency from kindergarten through secondary school.

Our students need to learn mathematics, and they need mathematics to learn.

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mean that we eliminate obstacles and challenges. Instead, we capitalize on the other strands of proficiency and provide opportunities for students to make mathematics meaningful through their perseverance and enthusiasm.

Proficiency develops over time, with practice, instruction, feedback, support, and through opportunity. As educators we take up the challenge of concurrently developing each of the strands of proficiency from kindergarten through secondary school. Becoming proficient in mathematics *is the start and end point*. For too long we have rested on incomplete definitions of the purpose of school mathematics and have engaged in misleading and distracting quarrels.

Our Full-Day Early Learning—Kindergarten program and the Ontario Mathematics Curricula for grades 1-12 both identify and promote proficiency. They provide the anchor and direction for mathematics instruction in Ontario. The expectations identify the classroom actions and the interconnectedness of the threads. *Explore, represent, design, justify, solve, compare* — these verbs direct the actions of proficiency. Our curriculum clearly identifies what proficiency looks like in a classroom. The work of educators is to design mathematics instruction that builds the strength of each thread in order to weave a rich and robust tapestry of proficiency.

Our students need to learn mathematics, and they need mathematics to learn. In order to elevate the discourse of mathematics education, our conversations must be rooted in proficiency because it is this aim toward excellence which will facilitate students to excel in their applications of mathematics and position them to realize its boundlessness.

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Dr. Chris Suurtamm—Planning Moves for Teachers (https://vimeo.com/136750780)

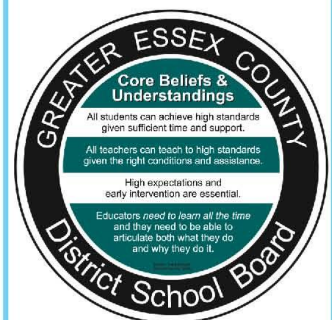
Dr. Cathy Fosnot—Basic Fact or Conceptual Understanding : A False Dichotomy (https://vimeo.com/104110510)

Dr. Cathy Fosnot—Conceptual Understanding and Procedural Fluency: We Need Both (https://vimeo.com/137299162)

GREATER ESSEX COUNTY DISTRICT SCHOOL BOARD

Supporting All Mathematics Learners through Responsive Learning Environments

The GECDSB has high expectations for all learners. We understand that learners come with diverse strengths, needs and interests. It is critical that we close the achievement gaps and meet the needs of all learners. Through a responsive mathematics learning environment, we can create equitable learning contexts and significantly impact student achievement. The GECDSB believes that we can design learning environments that are supportive, enriching and responsive to the uniqueness of our students.



The Greater Essex County District School Board provides mathematics education that engages and empowers students through collaboration, communication, inquiry, critical thinking, and problem-solving, to support each student's learning and nurture a positive attitude towards mathematics.

GECDSB, A Vision for Mathematics, 2016

The purpose of these learning briefs is to share the research, discussion and insight garnered from the intensive work of the Greater Essex County District School Board's Math Task Force. These papers are rooted in the GECDSB core beliefs, the Full-Day Early Learning—Kindergarten program and the Ontario Mathematics Curricula for grades 1-8, 9-10, and 11 & 12. The briefs are meant to elevate, enrich and extend the discourse of mathematics education with the intention of encouraging a positive and productive disposition toward mathematics for all learners.

Each paper provides a list of sources to extend the professional conversation and enhance the learning. In addition, live links appear at the end of the papers with connections to various resources.

MATH TASK FORCE

"All children can achieve high standards given sufficient time and support."

Effective mathematics teaching supports student learning according to the differentiated needs of the learner.

In order to support all learners, researchers insist that the learning environment must match the needs, strengths and readiness of the learner.

SUPPORTING ALL MATHEMATICS LEARNERS THROUGH RESPONSIVE LEARNING ENVIRONMENTS

The core beliefs of the Greater Essex County District School Board express the heart of teaching and learning. Our beliefs define our action, set our direction and determine our success. The GECDSB believes that "all children can achieve high standards given sufficient time and support." When we carefully examine the dimensions of "time and support" we are led to an understanding of the critical role that mathematics learning environments play in supporting all learners.

Understanding Achievement Gaps

Achievement gap is a widely used term which refers to the discrepancy in academic achievement of identifiable groups of students. *Learning for All: A Guide to Effective Assessment and Instruction for All Students (2013)* is an Ontario Ministry of Education publication which guides and supports school and system planning by addressing effective means of reaching all learners. It states:

> *Gaps in achievement can be measured in terms of various factors, such as gender, ethnocultural background, socio-economic status, special education needs, language proficiency, or number of credits accumulated by the end of a particular grade. Achievement gaps can also be defined according to combinations of these factors, such as gender and special education needs, or gender and socio-economic status, or ethnocultural background and credit accumulation by year and grade. (p. 11)*

Research suggests that background influences, specifically socio-economic status and parental involvement, play an important role in student achievement (Learning for All, 2013). These factors influence student *readiness to learn* and these levels of readiness are genuine realities of every classroom. Research also suggests that learning environments which are safe, supportive and meet the needs of students are a means of creating equitable contexts and have a significant impact on student achievement (Jensen, 1998; Marzano, 2003).

Certainly there are conditions over which educators have limited control. Our efforts are best applied, however, to those factors over which we do have significant influence. We can explicitly and methodically design learning environments that are supportive, enriching and responsive to the uniqueness of our students.

Effective mathematics teaching supports learning according to the differentiated needs of the student. In order to support *all* learners, researchers insist that the learning environment must match the needs, strengths and readiness of each student. In order to overcome the disparities in achievement, we must address *learning gaps,* which is a term "often used to refer to the gap between a student's actual achievement and his or her potential for achievement" (Learning for All, 2013). Our responsibility is to address these learning gaps with precision and intentionality.

Building Responsive Learning Environments

A learning environment includes the physical and social contexts in which students learn. These settings have a profound impact on the experiences of the learner. The Literacy and Numeracy Secretariat (2012) states that a responsive learning environment encompasses the physical and social-emotional environment which include such elements as student voice, collaborations, focus on solutions, real-world problem solving, and self-efficacy. The Department for Education and Skills in the UK explains that the learning environment is made up of three factors that overlap and impact each other: ethos, behaviours and routines, and the physical environment. Hannah (2013) identifies that the learning environment is affected by physical elements, emotional elements, and intangible elements such as the energy of the classroom, rules and sounds. Fraser (2012) defines the learning environment as

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"the social, psychological, and pedagogical contexts in which learning occurs and which affects student achievement and attitudes."

Based on the literature, it can be surmised that a responsive learning environment includes three realms: the Physical Realm, the Social and Emotional Realm, and the Choice and Voice Realm (Figure 1).

When considering the Physical Realm, educators look at the space of the classroom that promotes collaboration through group work as well as the space that permits quiet thinking and exploring of math. Active areas for inquiry, investigation and wonder are also considered to be part of the Physical Realm. When students are involved in the process of creating their own learning environment they can develop a sense of community and increased motivation.

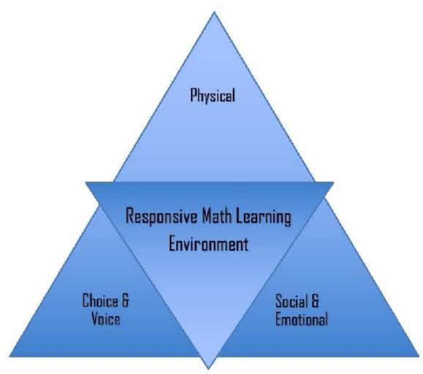
When educators work to foster the Social and Emotional Realm, students feel safer to take risks in math class so that they can make mistakes while trying new ideas and strategies. Students also feel safer to revise their ideas and develop new mathematical understandings. When students feel supported by educators, they develop a more positive attitude towards math. Students feel they learn better in a "togetherness" learning environment that provides them with a sense of community. Furthermore, when students have more opportunities to independently explore and inquire, they feel more connected to the classroom community. In these classrooms, students not only hold more positive attitudes toward math, but they demonstrate higher achievement in math (Yang, 2015).

Students need to feel they have a "choice and voice" in their learning. The Choice and Voice Realm is key in creating a responsive math learning environment that encourages diverse thinking perspectives and ideas that are valued as ways to deepen mathematical understanding. Students have choice in exploring mathematics by choosing tasks, tools, methods, and partners. Students need to engage in tasks that challenge their current understandings and therefore have multiple entry points to meet the needs (Suurtamm, Quigly, & Lazarus, 201; Boaler, 2015).

An effective mathematics learning environment is determined by many factors and is influenced by all levels of educational organizations. Thus, decisions at all levels and by all stakeholders must take heed of the impact of these on the learning environment.

Each and every school day our classrooms are filled with students who bring the uniqueness of their individual stories to their learning. Each and every day we work together to write the pages of these stories. Planning these settings with purpose and intentionality is one way in which we can constructively shape the success of the stories our students will tell.

Figure 1



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3

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LINKS

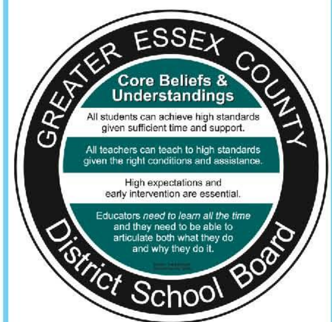
Guides to Effective Instruction
http://www.edugains.ca/newsite/math/guides_effective_instruction.html

The Third Teacher
http://www.edugains.ca/resourcesLNS/Monographs/CapacityBuildingSeries/CBS_ThirdTeacher.pdf

Describing an Effective Learning Environment
http://www.edugains.ca/newsite/math/learningenvironments.html

GREATER ESSEX COUNTY
DISTRICT SCHOOL BOARD

The Basics about Back to Math Basics



The GECDSB believes that the current challenge for educators is teaching mathematics that is rooted in, but not limited to, the "basics." Although mathematics has a practical value as a life skill, it is also applied to an array of other disciplines and is a creative and abstract science of numbers, space, and relationships between objects. The current paradigm of mathematics education must to be grounded in a definition of *mathematical proficiency* and our conversations need to extend beyond fundamentals and aim toward excellence.

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MATH
TASK FORCE

Basic understanding of mathematics is essential, and a necessary function of our schools is to prepare children with these foundational concepts and skills.

2

THE BASICS ABOUT BACK TO MATH BASICS

We agree: math is everywhere and plays an important role in our lives. Basic understanding of mathematics is essential, and a necessary function of our schools is to prepare children with these foundational concepts and skills. The National Research Council's publication, *Adding it Up: Helping Children Learn Math,* articulates that, "(mathematics) is so much a part of modern life that anyone who wishes to be a fully participating member of society must know basic mathematics" (NCTM, 2001, p.15). In addition, a range of literature has demonstrated that early numeracy skills are a significant predictor of later academic achievement: perhaps even greater than early literacy skills (Duncan, Dowsett, Claessens, Magnuson, Huston, Klebanov, Pagani, Feinstein, Engel, Brooks-Gunn, Sexton, Duckworth, & Japel, 2007).

There is little argument that success in mathematics is imperative for 21st century learners, and most would agree that proficiency implies a foundation of basic mathematics. Yet the 'back to basics' argument continues in popular media, distracting public discourse from the deeper and significantly more complex issues of 21st century mathematics teaching and learning. Although teaching 'the basics' is a function of our schools, the real challenge for educators is teaching mathematics that is rooted in, but not limited to, 'the basics'. The current paradigm of mathematics education must be grounded in a definition of *mathematics proficiency* and our conversations need to extend beyond fundamentals and aim toward excellence.

Defining Mathematical Proficiency

The experience that we understand best is our own. Our experiences with mathematics are shaped by our learning and in turn our learning shapes our experiences. Depending on who you ask, the question of 'what it means to do math' is answered three different ways and each of these explanations has implications for the teaching and learning of mathematics. The first description refers to mathematics as an everyday skill. From basic banking to baking, we use mathematics to function in our everyday living. Although mathematics certainly has a practical value, there is significant danger in only defining mathematics as a 'life skill'. We do not define literacy as having a merely functional value. We expect students to develop an array of literacy skills that exceed a basic level. We believe that students should have access to rich texts with vivid discourse and critical thought. Why would we lower the bar for mathematics?

Mathematics is also distinct for its application to other disciplines, trades and professions. Modern areas of applied mathematics include medicine, business, technology, engineering and architecture as well as the sciences and others. Applied mathematics is not only relevant to most professions, it is crucial. People know and understand the math they experience in their professional lives, and sometimes reflect on this application as a definition of what it means to use and be skilled in mathematics.

Although both of these explanations are dimensions of math, they do not encompass the sum of mathematics as a creative, esoteric, abstract science of number and space. Mathematics is a rich and diverse discipline that dates to the beginning of recorded history. The goal of mathematics education should comprise this depth and richness. The Ontario Mathematics Curriculum states that:

Learning mathematics results in more than a mastery of basic skills. It equips students with a concise and powerful means of communication. Mathematical structures, operations, processes, and language provide students with a framework and tools for reasoning, justifying conclusions, and expressing ideas clearly (The Ontario Curriculum Grades 1-8: Mathematics, 2005).

The History of School Mathematics

The question of 'what should be taught in schools' has sparked debate by education philosophers since the time of Plato. During the first half of the 20th century, mathematics skill focused on computational procedures of arithmetic. In the 1950s and 1960s, the focus of mathematics education began to shift to include the structures of mathematics. From this shift was born the 'back to basics' movement which is described in *Adding it Up*, as "a returning to the view that success in mathematics meant being able to compute accurately and quickly" (National Research Council, 2001, p. 115). This view of mathematics as simply arithmetic computations is incomplete and incompatible with the modern goals of education. Basic skills of mathematics are developed in service of the richness of mathematics, but they are not its entirety.

The authors of, *Adding it Up* indicate that "recognizing that no one term captures completely all aspects of expertise, competence, knowledge and facility in mathematics, we have chosen *mathematical proficiency* to capture what we believe is necessary for anyone to learn mathematics successfully." The Greater Essex County District School Board's, Vision for Mathematics is based on the extensive evidence in mathematics education research that identifies five interdependent skill-sets which work together in the development of mathematical proficiency (Figure 1). Each of these strands represents distinct and important aspects of the whole of what it means to be proficient in mathematics.

Mathematics for 21st Century Learning

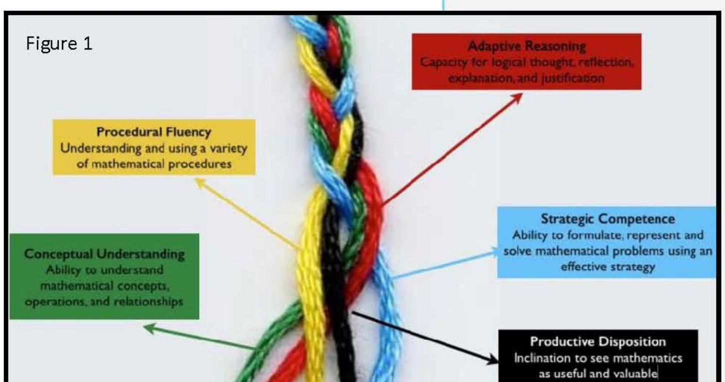
In order to inform practices and policies for school mathematics it is imperative that we look beyond our personal experience and critically examine evidence provided by research. Our Ontario Mathematics Curriculum grounds us in this conversation and states:

An information- and technology-based society requires individuals who are able to think critically about complex issues, analyse and adapt to new situations, solve problems of various kinds, and communicate their thinking effectively. The study of mathematics equips students with knowledge, skills, and habits of mind that are essential for successful and rewarding participation in such a society. To learn mathematics in a way that will serve them well throughout their lives, students need classroom experiences that help them develop mathematical understanding; learn important facts, skills, and procedures; develop the ability to apply the processes of mathematics; and acquire a positive attitude towards mathematics. (The Ontario Curriculum Grades 1-8: Mathematics, 2005)

We educate for excellence! We must extend our discourse to honour the essence of mathematics because in every classroom sits an artist, writer, builder, scientist and mathematician. It is the fullness of the educational experience that reaches the heart of every learner. Our purpose as educators is not to count the limited prospects but to inspire the infinite possibilities.

If you deny students the opportunity to engage in this activity – to pose their own problems, to make their own conjectures and discoveries, to be wrong, to be creatively frustrated, to have an inspiration, to cobble together their own explanations and proofs – you deny them mathematics itself.

(Lockhart, 2009)



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LINKS

MARIAN SMALL—THE ART OF MATHEMATICS
https://vimeo.com/103956482

CATHY FOSNOT—BASIC FACTS OR CONCEPTUAL UNDERSTANDING: WE NEED BOTH
https://vimeo.com/104110510

DAN MEYER—MATH CLASS NEEDS A MAKEOVER
http://www.ted.com/talks/dan_meyer_math_curriculum_makeover

CHRIS SUURTAMM—PLANNING MOVES FOR TEACHERS
https://vimeo.com/136750780

CATHY BRUCE—MATH TALK
https://www.youtube.com/watch?v=ypIXNE4PRQQ

CREATING CONDITIONS FOR MATHEMATICS LEARNING
http://www.curriculum.org/k-12/en/projects/creating-the-conditions-for-learning-mathematics

DAN MEYER—REAL WORLD MATH
https://www.youtube.com/watch?v=jRMVjHjYB6w

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